Part 2: Gödel's Proof of the Existence of God

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Review: Intensional vs. Exstensional Objects

Extensional Object: a set or relation in the usual sense

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Example:

- Possible worlds are people, the domain as real-world objects
- each person will classify some of those objects as being *red* (type $\langle 0 \rangle$).
- The *red concept* maps to each person the set of objects he/she considers red (type $\uparrow\langle 0 \rangle$).
- The *color concept* maps to each person the set of *color* (concepts) for that person (type $\uparrow\langle\uparrow\langle 0 \rangle\rangle$)

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1. $\forall x \Diamond T(x)$
2. $\Diamond \forall x T(x)$
3. But do we mean, "tall" as we currently use the word tall, or as the word is used in the future?

(x: type 0, P: type $\uparrow\langle 0\rangle$, X: type $\uparrow\langle 0\rangle$)

$\langle\lambda X.\Diamond(\exists x)X(x)\rangle(P) \leftrightarrow \Diamond\langle\lambda X.(\exists x)X(x)\rangle(P)$ is valid

$\mathcal{M}, \Gamma \models_v \langle\lambda X.\Diamond(\exists x)X(x)\rangle(P)$

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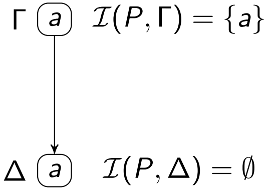

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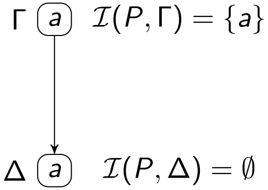

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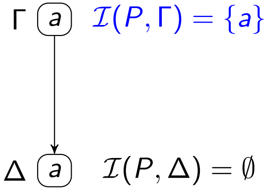

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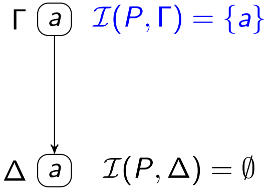

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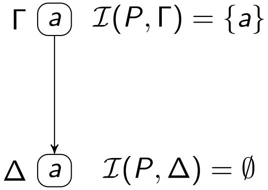

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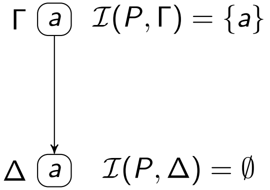

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Tableaus

Possibly God exists

Informal Axiom 1: Exactly one of a property or its complement is positive

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Definition: P **entails** Q if, necessarily, everything having P also has Q.

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Informal Axiom 1: Exactly one of a property or its complement is positive

Definition: P **entails** Q if, necessarily, everything having P also has Q.

Informal Axiom 2: Any property entailed by a positive property is positive

Informal Proposition 1: Any positive property is possibly instantiated. I.e., if P is positive then it is possible that something has property P.

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Informal Axiom 3: The conjunction of any collection of positive properties is positive.

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God's existence is necessary, if possible

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1. g has property G
2. G entails every property of g

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Informal Proposition: If g is a God, the essence of g is being a God.

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Informal Proposition If a God exists, a God exists necessarily.

Informal Proposition If it is possible that a God exists, it is necessary that a God exists (assume **S5**)

Informal Theorem Assuming all the axioms, and assuming that the underlying logic is **S5**, a (the) God necessarily exists.

Formalizing Proposition 1

Definition: Let $\mathcal{P}$ represent **positiveness**. $\mathcal{P}$ is a constant symbol of type $\uparrow\langle\uparrow 0\rangle$. P is positive if we have $\mathcal{P}(P)$.

Definition If τ is a term of type $\uparrow\langle 0\rangle$, take $\neg\tau$ as short for $\langle\lambda x.\neg\tau(x)\rangle$. Call τ negative if $\neg\tau$ is positive.

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Formalizing Axiom 1 (Axiom 11.3)

1. $\forall X[\mathcal{P}(\neg X) \to \neg\mathcal{P}(X)]$
2. $\forall X[\neg\mathcal{P}(X) \to \mathcal{P}(X)]$

Formalizing Proposition 1

Formalizing Axiom 2 (Axiom 11.5)

$$(\forall X)(\forall Y)[[\mathcal{P}(X) \wedge \Box(\forall^{\mathsf{E}} x)(X(x) \rightarrow Y(x))] \rightarrow \mathcal{P}(Y)]$$

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Proposition Assuming 11.5

1. $(\exists X)\mathcal{P}(X) \rightarrow \mathcal{P}(\langle \lambda x.x = x \rangle)$
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Formalizing Informal Proposition 1 Assuming 11.3 A and 11.5

$$(\forall X)[\mathcal{P}(X) \rightarrow \Diamond(\exists^{\mathsf{E}} x)X(x)]$$

Formalizing Informal Axiom 3

Axiom 11.9: $(\forall X)(\forall Y)[[\mathcal{P}(X) \wedge \mathcal{P}(Y)] \rightarrow \mathcal{P}(X \wedge Y)]$

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But this should hold for any number of Xs

1. $\mathcal{Z}$ applies to only positive properties:

$$pos(\mathcal{Z}) \ := \ (\forall X)[\mathcal{Z}(X) \rightarrow \mathcal{P}(X)]$$

2. X is the (necessary) intersction of $\mathcal{Z}$

$$(X \ intersection \ of \ \mathcal{Z}) \ := \ \Box(\forall x)[X(x) \leftrightarrow (\forall Y)[\mathcal{Z}(Y) \rightarrow Y(x)]]$$

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Technical Assumptions (Axiom 4)

$$(\forall X)[\mathcal{P}(X) \rightarrow \Box\mathcal{P}(X)]$$

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Axiom 11.11: $(\forall X)[\mathcal{P}(X) \to \Box\mathcal{P}(X)]$.

Being Godlike

Godlike is an intension term of type $\uparrow\langle 0\rangle$, intuitively the set of god-like objects at a world.

Definition 11.12 G is the following type $\uparrow\langle 0\rangle$ term:

$$\langle\lambda x.(\forall Y)[\mathcal{P}(Y) \rightarrow Y(x)]\rangle$$

Definition 11.13 G^* is the following type $\uparrow\langle 0\rangle$ term:

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Proposition Assuming 11.3B, in **K**, $(\forall x)[G(x) \leftrightarrow G^*(x)]$.

Possibly God exists

Theorem 11.17 Assume axioms 11.3A, 11.5 and 11.10. In **K** both of the following are consequences: $\Diamond(\exists^{E}x)G(x)$ and $\Diamond(\exists x)G(x)$.

Objection 1

Theorem Assume all the axioms except for 11.10 and 11.9, the following are equivalent using **S5**:

1. Axiom 11.10:
   $(\forall \mathcal{Z})[pos(\mathcal{Z}) \rightarrow \forall X[(X \ intersection\ of\ \ \mathcal{Z}) \rightarrow \mathcal{P}(X)]$
2. $\mathcal{P}(G)$
3. $\Diamond(\exists^{\mathbf{E}} x)G(x)$

Necessarily God exists

Formalizing Informal Definition 6 Let N abbreviate the following type $\uparrow\langle 0\rangle$ term:

$$\langle \lambda x.(\forall Y)[\mathcal{E}(Y,x) \rightarrow \Box(\exists^{\mathsf{E}} z Y(z))]\rangle$$

something has property N of necessary existence provided any essence of it is necessarily instantiated.

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something has property N of necessary existence provided any essence of it is necessarily instantiated.

Axiom 11.25: $\mathcal{P}(N)$.

Essence

The **essence** of something, x, is a property that *entails* every property that x possesses: Intuitively,

$$(\varphi \text{ Ess } x) \ \leftrightarrow \varphi(x) \wedge (\forall\psi)[\psi(x) \rightarrow \Box\forall y[\varphi(y) \rightarrow \psi(y)]$$

Definition $\mathcal{E}$ abbreviates the following $\uparrow\langle\uparrow\langle 0\rangle, 0\rangle$, term ($Z$ is type $\uparrow\langle 0\rangle$ and w is type 0):

$$\langle\lambda Y, x.Y(x) \wedge \forall Z[Z(x) \rightarrow \Box(\forall^{\mathsf{E}} w)[Y(w) \rightarrow Z(w)]]\rangle$$

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Theorem Assume axioms 11.3B and 11.11, in **K** the following is provable: $(\forall x)[G(x) \rightarrow \mathcal{E}(G, x)]$ (same for G^*).

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Theorem In **K**, the following is provable

$$(\forall X)(\forall y)[\mathcal{E}(X, y) \rightarrow \Box(\forall^{\mathsf{E}} z[X(z) \rightarrow (y = z)]]$$

Necessarily God exists

Theorem Assume Axioms 11.3B, 11.11, 11.25, in **K**

$$(\exists x)G(x) \to \Box(\exists^{\mathsf{E}} x)G(x)$$

Necessarily God exists

Theorem Assume Axioms 11.3B, 11.11, 11.25, in **K**

$$(\exists x)G(x) \to \Box(\exists^{\mathsf{E}} x)G(x)$$

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Corollary $\Box(\exists^{\mathsf{E}} x)G(x)$

Conclusions

- Other objections: the modal system collapses ($Q \rightarrow \Box Q$ is valid)
- Fitting has a number of papers which develops and applies (fragments of) this framework (papers on Database Theory, logics "between" propositional and first order.
- ....