Logics of Rational Agency

Lecture 4

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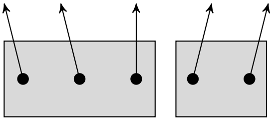
- $[\mathit{stit}]\varphi$: the agent can "see to it that" φ is true
- $\Diamond[\mathit{stit}]\varphi$: the agent has the ability to bring about φ

Many Agents

The previous model assumes there is *one* agent that "controls" the transition system.



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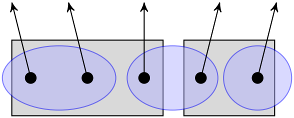

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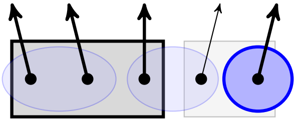

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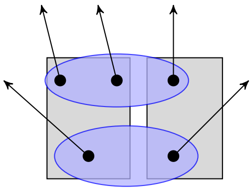

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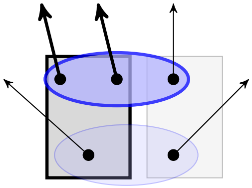

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From Temporal Logic to Strategy Logic

- *Coalitional Logic*: Reasoning about (local) group power.

  $[C]\varphi$: coalition C has a **joint action** to bring about φ.

M. Pauly. *A Modal Logic for Coalition Powers in Games*. *Journal of Logic and Computation* **12** (2002).

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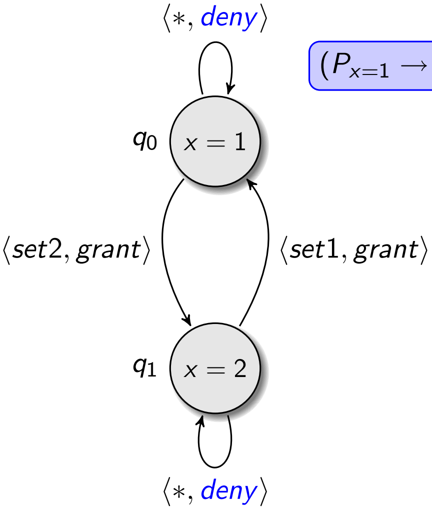
- *Alternating-time Temporal Logic*: Reasoning about (local and global) group power:

  $\langle\langle A\rangle\rangle\Box\varphi$: The coalition A has a **joint action** to ensure that φ will remain true.

R. Alur, T. Henzinger and O. Kupferman. *Alternating-time Temproal Logic*. *Jouranl of the ACM* (2002).

Multi-agent Transition Systems



$(P_{x=1} \rightarrow [s]P_{x=1}) \wedge (P_{x=2} \rightarrow [s]P_{x=2})$

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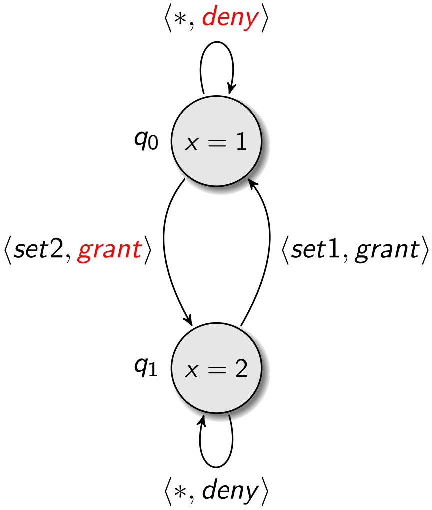

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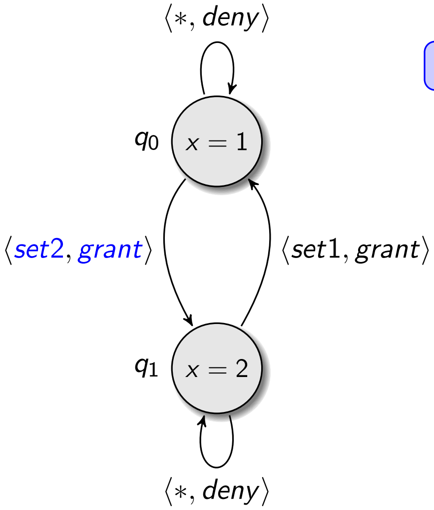

$P_{x=1} \longrightarrow \neg[s, c]P_{x=2}$

Logics of Abilities

A. Herzig and F. Schwarzentruber. *Properties of logics of individual and group agency*. C. Areces and R. Golblatt (eds.) Advances in Modal Logic, pgs. 133 - 149, 2008.

A. Herzig and E. Lorini. *A Dynamic Logic of Agency I: STIT, Capabilities and Powers*. Journal of Logic, Language and Information, 19, pgs. 89 - 121, 2010.

End of Part 2.

General Issues

Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.

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How should we *compare* the different logical systems?

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- Comparing different frameworks: eg. PDL vs. Temporal Logic, PDL vs. STIT, STIT vs. ATL, etc.

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Theorem $\Box\varphi \leftrightarrow \varphi$ is provable in combinations of Epistemic Logics and PDL with certain "cross axioms" ($\Box[a]\varphi \leftrightarrow [a]\Box\varphi$) (and full substitution).

R. Schmidt and D. Tishkovsky. *On combinations of propositional dynamic logic and doxastic modal logics*. JOLLI, 2008.

Merging Logics of Rational Agency

- Entangling Knowledge/Beliefs and Preferences
- Reasoning with *protocols*
- "Epistemizing" Logics of Action and Ability: *knowing how to achieve* φ vs. *knowing that you can achieve* φ
- Plans Change

Logics of Knowledge and Preference

$K(\varphi \succeq \psi)$: "Ann knows that φ is at least as good as ψ"

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J. van Eijck. *Yet more modal logics of preference change and belief revision*. manuscript, 2009.

F. Liu. *Changing for the Better: Preference Dynamics and Agent Diversity*. PhD thesis, ILLC, 2008.

$$A(\psi \rightarrow \langle \succeq \rangle \varphi) \quad \textit{vs.} \quad K(\psi \rightarrow \langle \succeq \rangle \varphi)$$

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$\mathcal{M}, w \models \langle\succeq \cap \sim\rangle\varphi$ iff there is a v with $w \sim v$ and $w \preceq v$ such that $\mathcal{M}, v \models \varphi$

$K(\psi \to \langle\succeq \cap \sim\rangle\varphi)$

Defining Beliefs from Preferences

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- Typically the results come in the form of a representation theorem:

  > *If the agents preferences satisfy such-and-such properties, then there is a set of conditional probability functions and a (state independent) utility function such that the agent can be assumed to act as an* expected utility maximizer.

Thus logical properties of beliefs can be derived from properties of preferences.

S. Morris. *The Logic of Belief and Belief Change: A Decision Theoretic Approach*. Journal of Economic Theory (1996).

The Framework

Let Ω be a set of states.

An **act** is a function $x : \Omega \to \mathbb{R}$. Let $\Re^\Omega$ be the set of all acts.

x_w for $w \in \Omega$ means that if the true state is w, then the agent receives prize x.

We write $x \succeq_w y$ the agent prefers x over y *provided the true state is w*

Belief Operators

A **belief operator** is a function $B : 2^{\Omega} \to 2^{\Omega}$

For $E \subseteq \Omega$, $w \in B(E)$ means the agent believes E at state w

B is normal if

- $B(\Omega) = \Omega$
- $B(E \cap F) = B(E) \cap B(F)$

Possibility function: $P : \Omega \to 2^{\Omega}$: set of states the agent considers possible at w

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B **reflects** $\{\succeq_w\}_{w\in\Omega}$ provided for each $E \subseteq \Omega$

$$B(E) = \{w \mid (x_E, y_{-E}) \sim_w (x_E, z_{-E}) \quad \text{for all } x, y, z \in \Re^\Omega\}$$

Theorem If the preference relations are complete and transitive, then the derived belief operator is normal.

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3. Logical issues: language design, axiomatization issues

Epistemic Dynamics and Protocols

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Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

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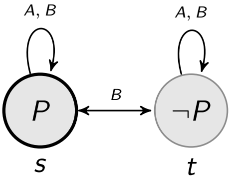
Is this procedure correct? Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

Example



P means "The talk is at 2PM".

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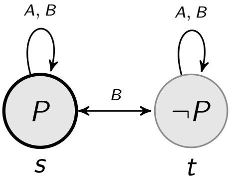

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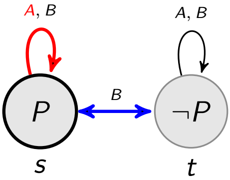

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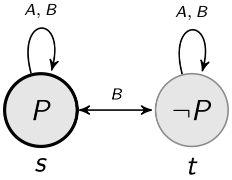

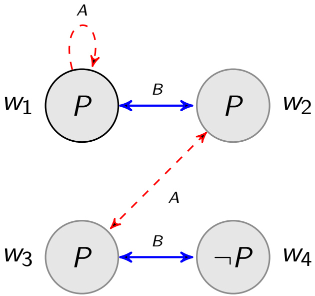

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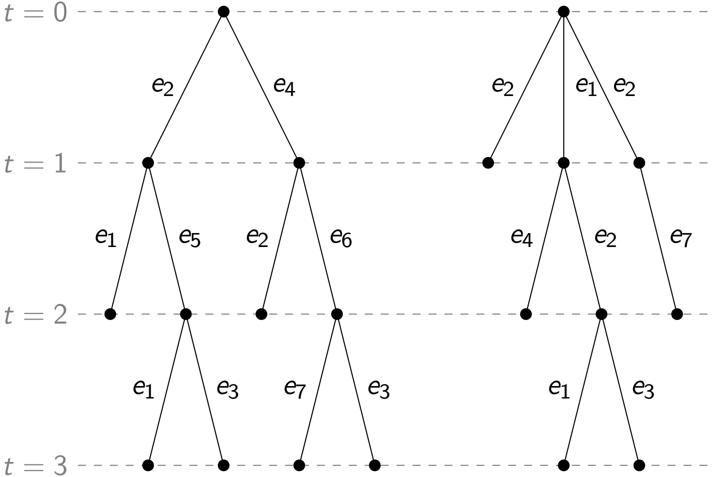
Dynamic Epistemic Logic

Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The ‘Playground’



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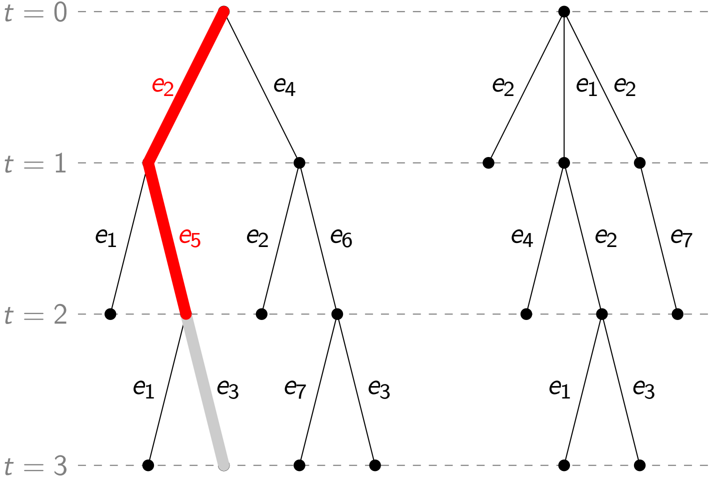

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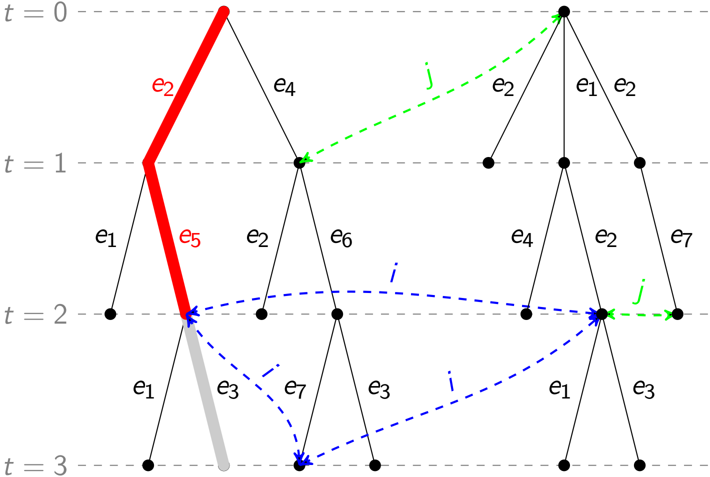

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \rightarrow 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

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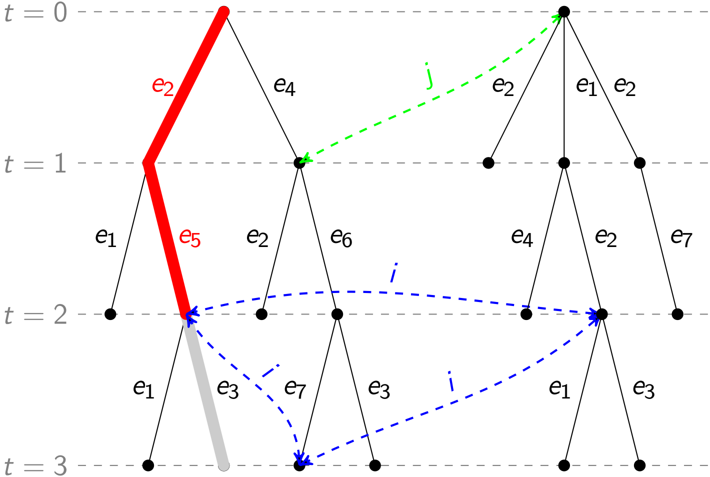
t = 0
t = 1
t = 2
t = 3
e2
e4
j
e2
e1
e2
e1
e5
e2
e6
i
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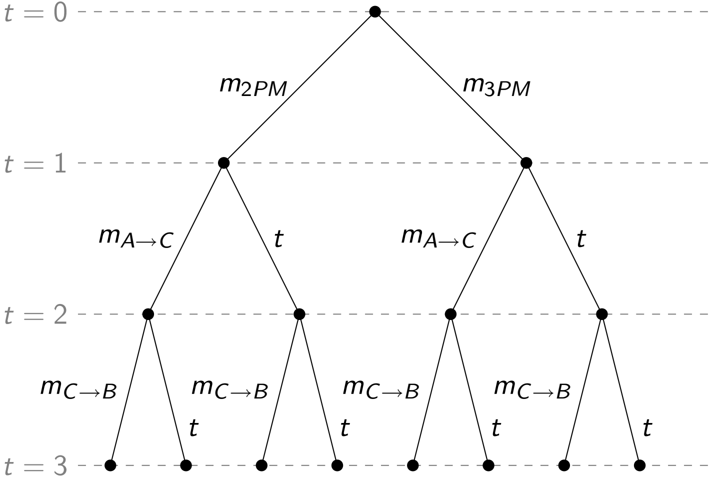
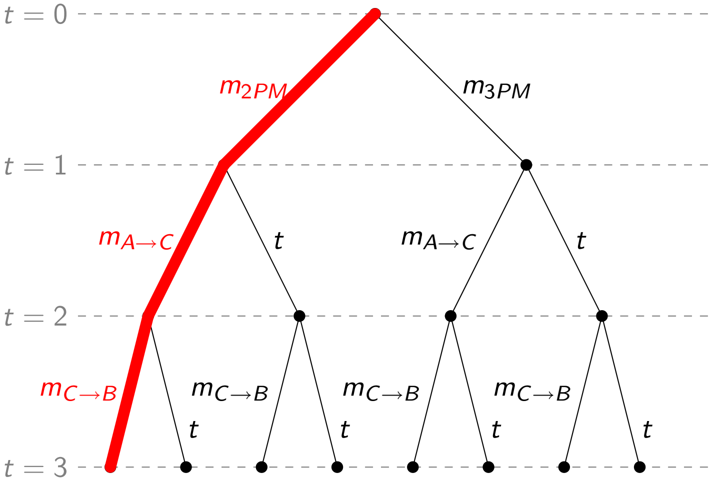
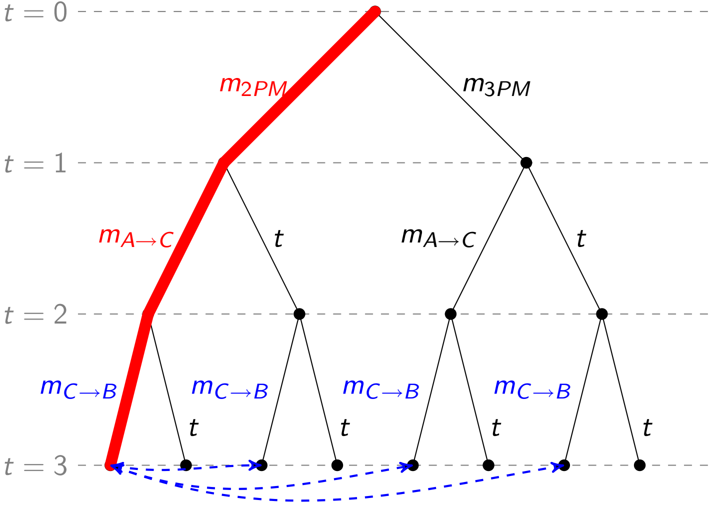
There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

Is this procedure correct?

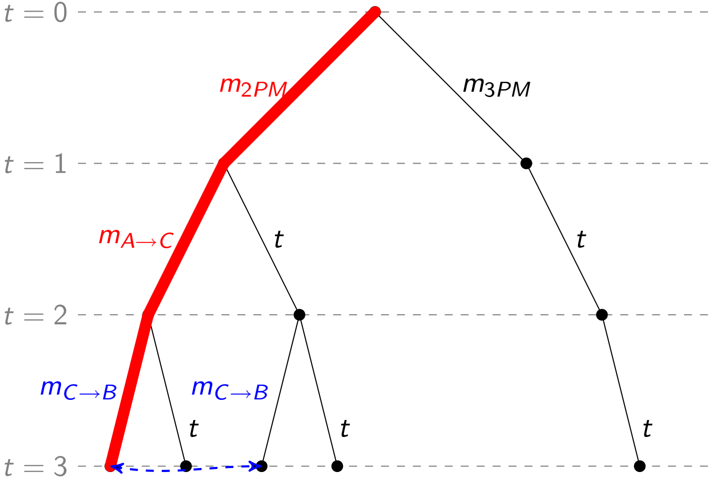

t = 0
t = 1
t = 2
t = 3
m_{2PM}
m_{3PM}
m_{A→C}
t
m_{A→C}
t
m_{C→B}
t
m_{C→B}
t
m_{C→B}
t
m_{C→B}
t



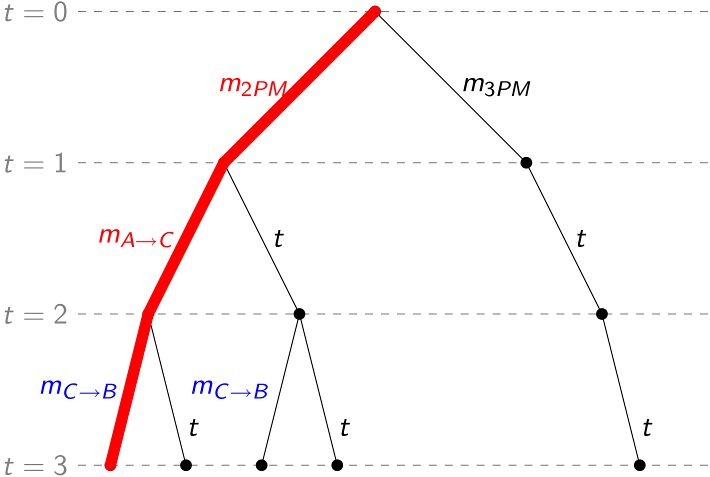
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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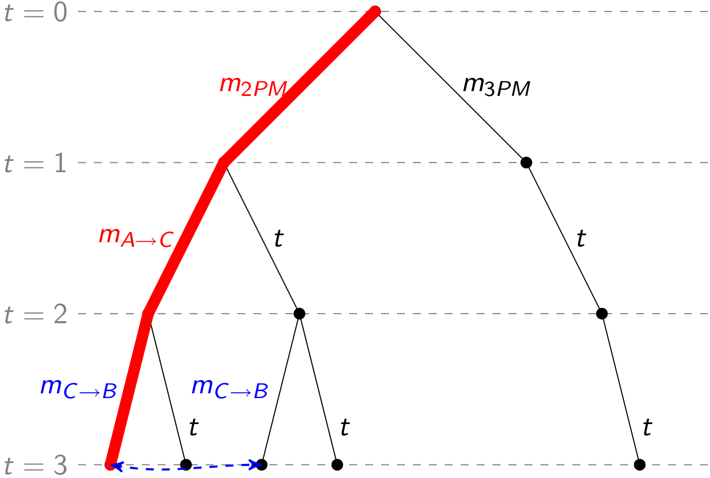

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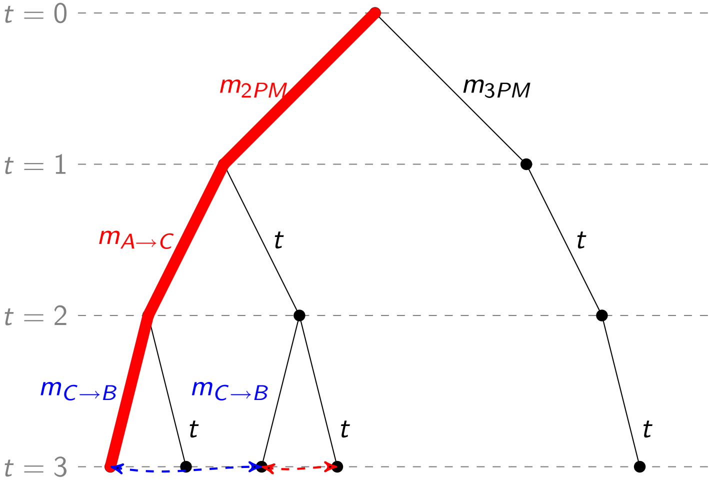

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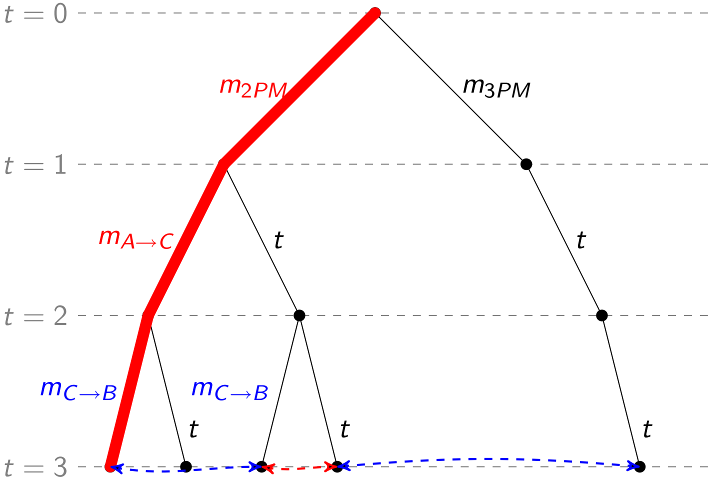

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Parameters of the Logical Framework

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1. Expressivity of the formal language. Does the language include a common knowledge operator? A future operator? Both?

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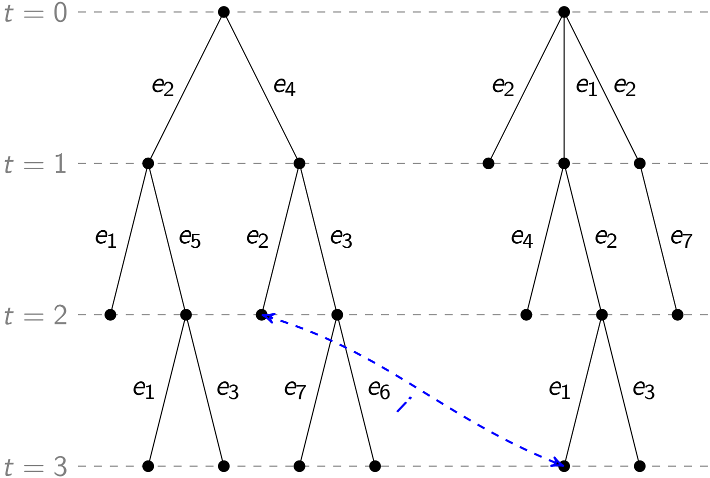
3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

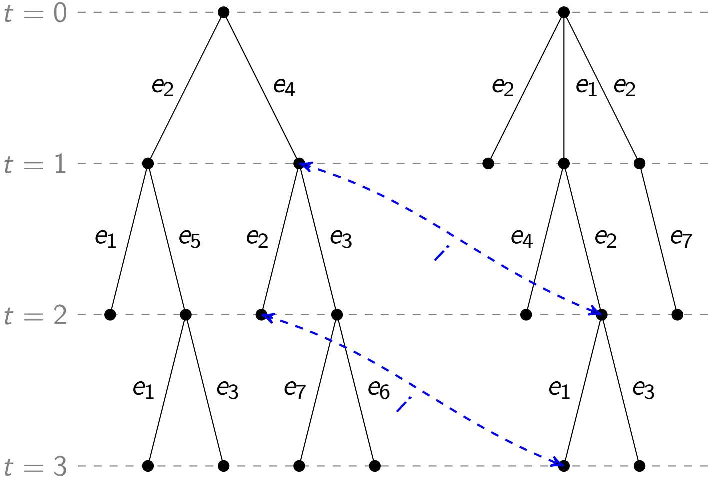
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.
- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.
- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\mathrm{len}(H) = \mathrm{len}(H')$.

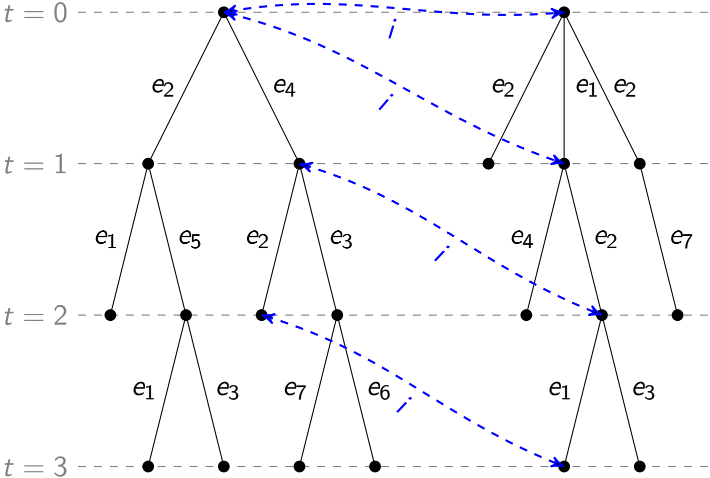
Perfect Recall



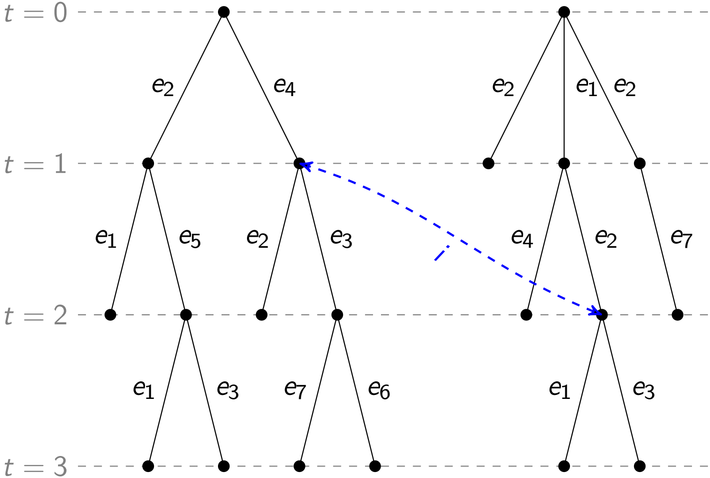
Perfect Recall



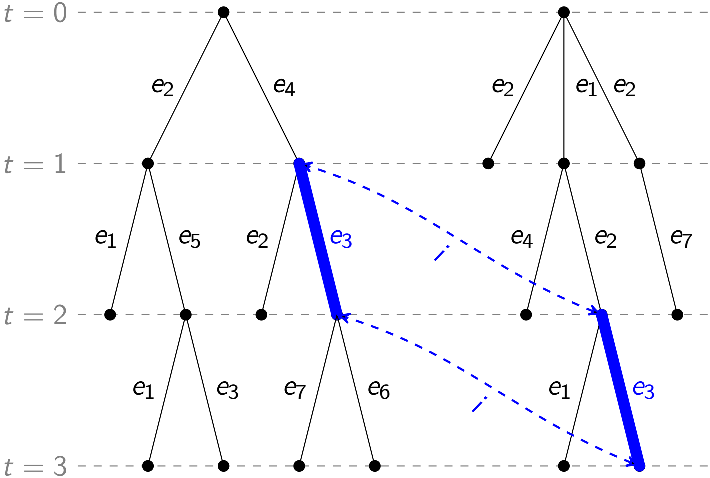
Perfect Recall



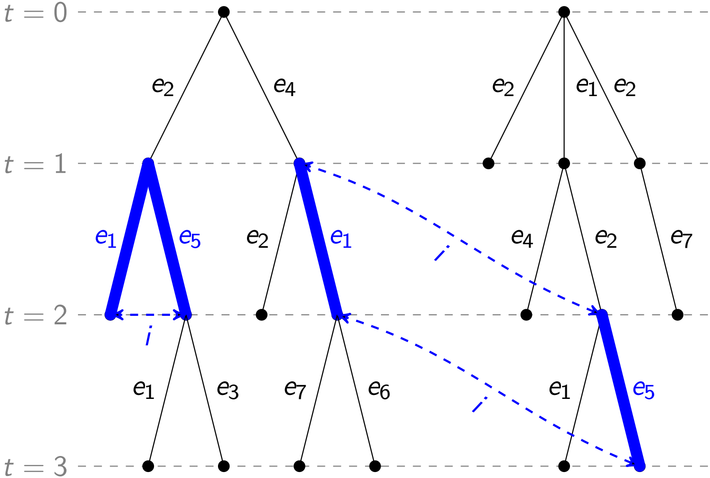
No Miracles



No Miracles



No Miracles



Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

Two Methodologies

ETL methodology: when describing a social situation, first write down all possible sequences of events, then at each moment write down the agents' uncertainty, from that infer how the agents' knowledge changes from one moment to the next.

Alternative methodology: describe an initial situations, provide a method for how events change a model that can be described in the formal language, then construct the event tree as needed.

Dynamic Epistemic Logic

Returning to the Example: DEL

Returning to the Example: DEL

$$(\mathcal{M} \otimes E_1) \otimes E_2$$

Returning to the Example: DEL

$$(\mathcal{M} \otimes E_1) \otimes E_2$$

The initial model (Ann and Bob are ignorant about P_{2PM}).

Private announcement to Ann about the talk.

Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

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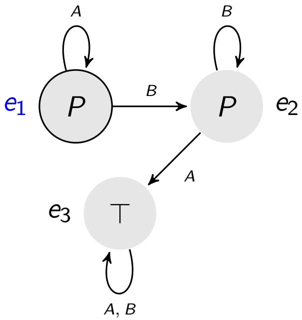

Abstract Description of the Event

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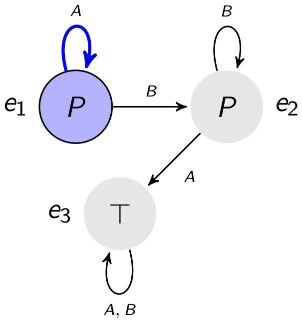
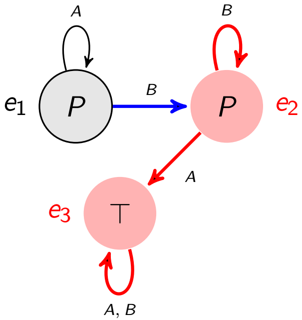


Ann knows which event took place.

Abstract Description of the Event

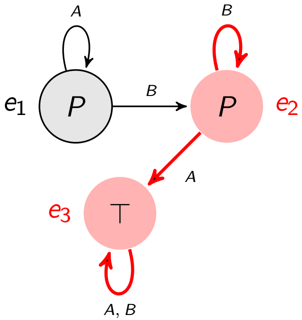
Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.



Bob thinks a different event took place.

Abstract Description of the Event

Recall the Ann and Bob example: Charles tells Bob that the talk is at 2PM.

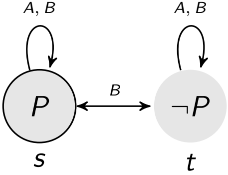


That is, Bob learns the time of the talk, but Ann learns nothing.

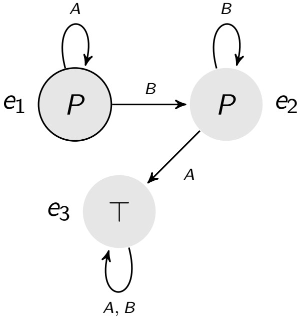
Product Update

Product Update

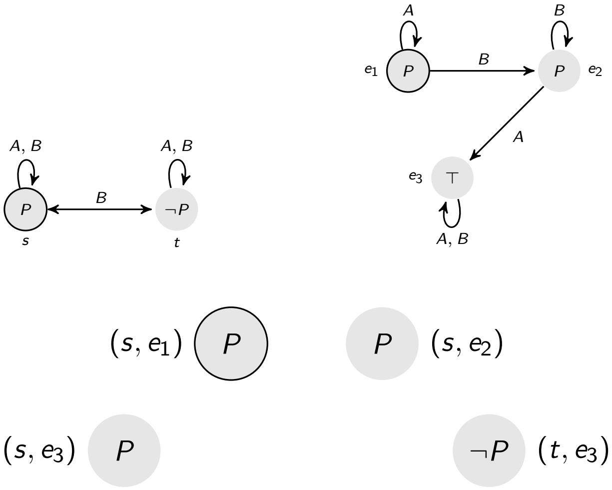
$\mathcal{M} \otimes E_1$



E_2



Product Update



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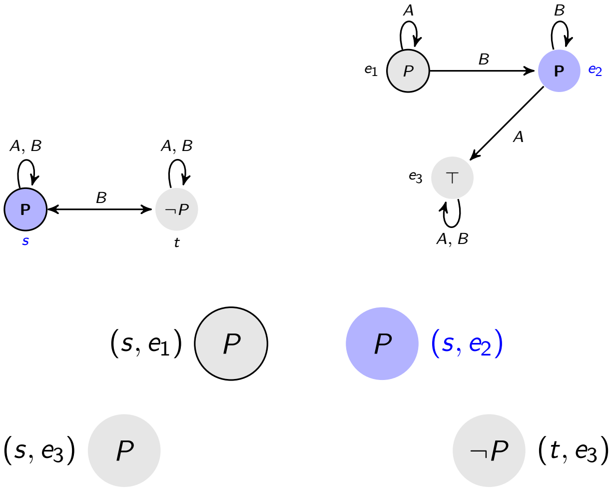

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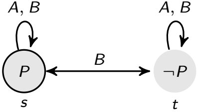
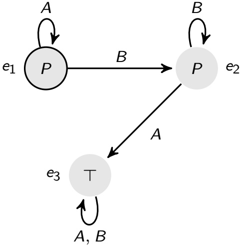
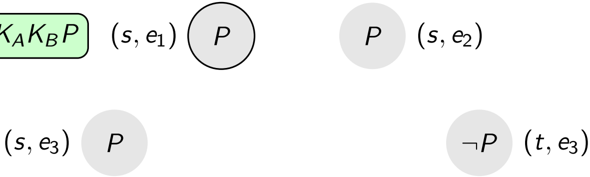




$(s, e_1) \models \neg K_B K_A K_B P$



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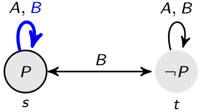

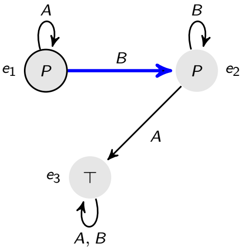

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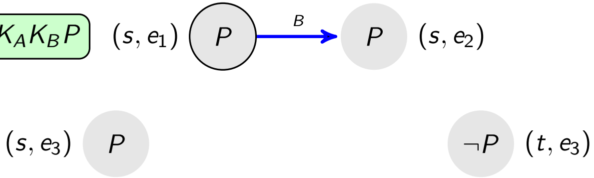

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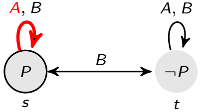

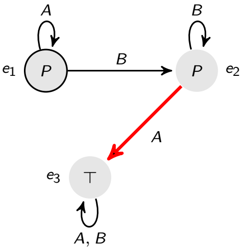

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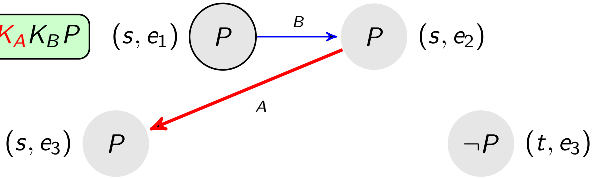
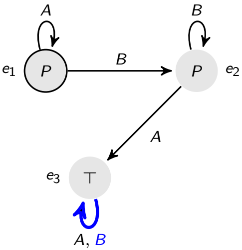
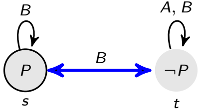


Product Update





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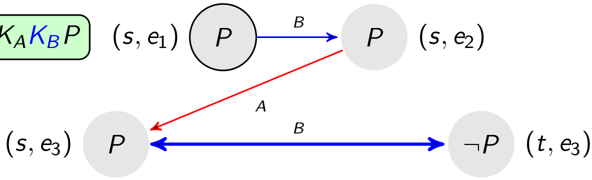

Product Update Details

Let $\mathbb{M} = \langle W, R, V \rangle$ be a Kripke model.

An event model is a tuple $\mathbb{A} = \langle A, S, Pre \rangle$, where $S \subseteq A \times A$ and $Pre : \mathcal{L} \rightarrow \wp(A)$.

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$\mathcal{M}, w \models [\mathrm{A}, a]\varphi$ iff $\mathcal{M}, w \models Pre(a)$ implies $\mathcal{M} \otimes A, (w, a) \models \varphi$.

Literarture

A. Baltag and L. Moss. *Logics for Epistemic Programs*. 2004.

W. van der Hoek, H. van Ditmarsch and B. Kooi. *Dynamic Episetmic Logic*. 2007.

DEL *and* ETL

Observation: By repeatedly updating an epistemic model with event models, the machinery of DEL creates ETL models.

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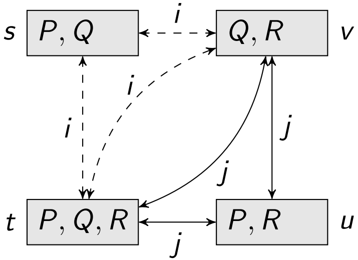
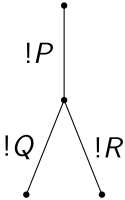
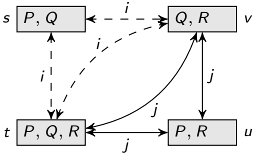
Let M be an epistemic model, and P a DEL protocol (tree of event models). The ETL model generated by M and P, forest(M, P), represents all possible evolutions of the system obtained by updating M with sequences from P.

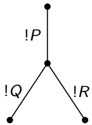
Example: Initial Model and Protocol





Example





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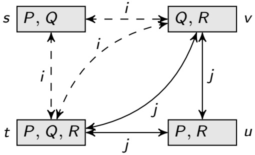
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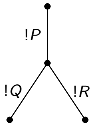
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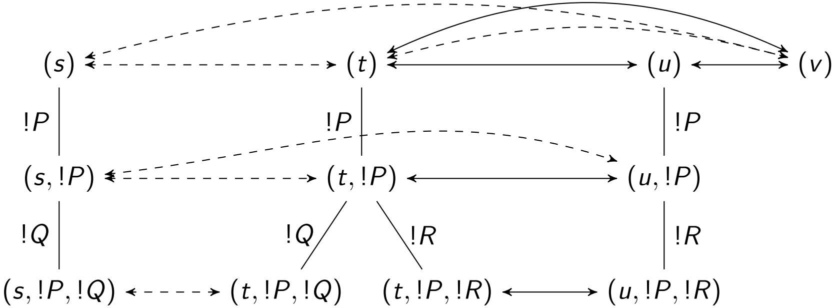
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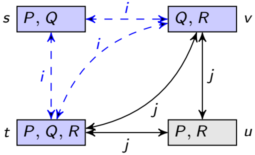
Example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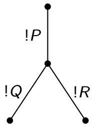
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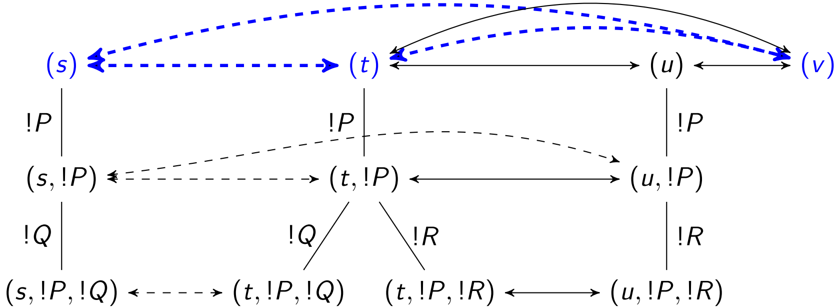
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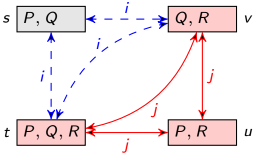
(u, !P, !R)

Example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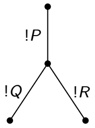
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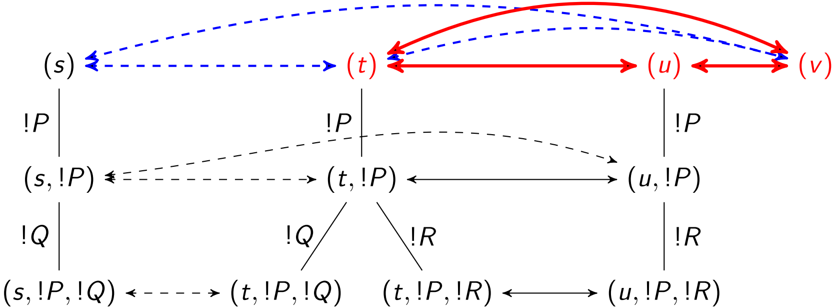
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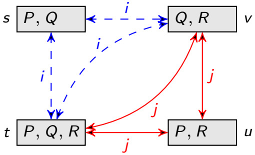
Example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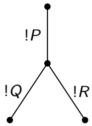
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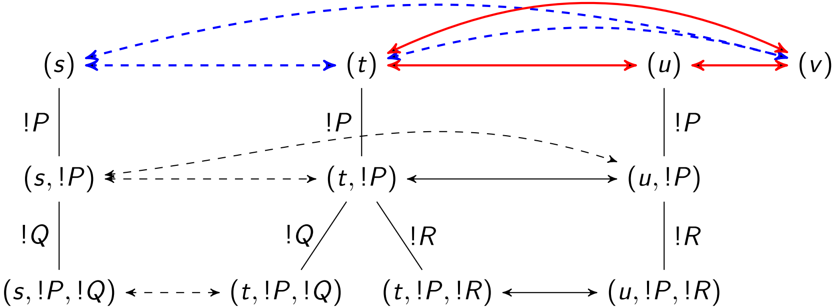
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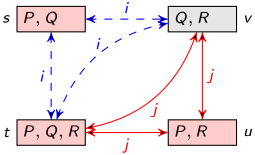
Example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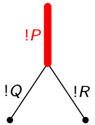
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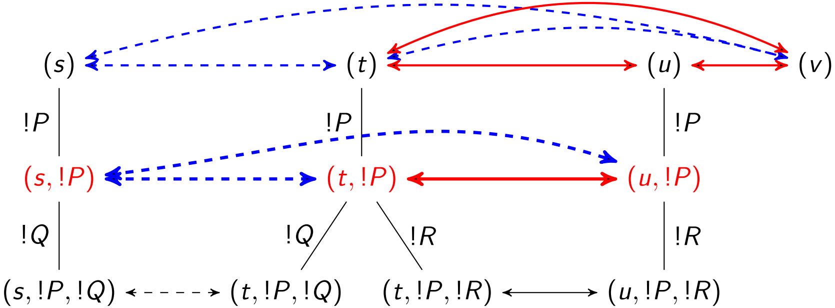
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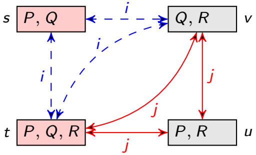
Example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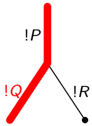
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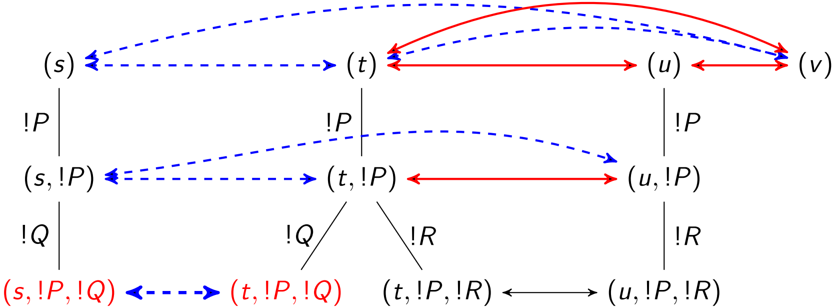
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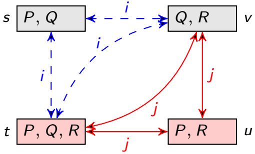
Example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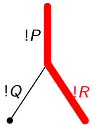
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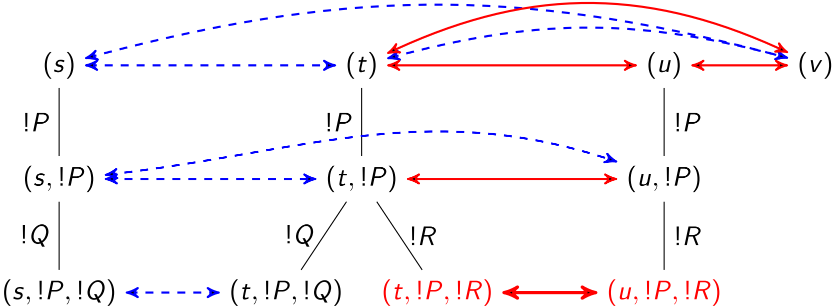
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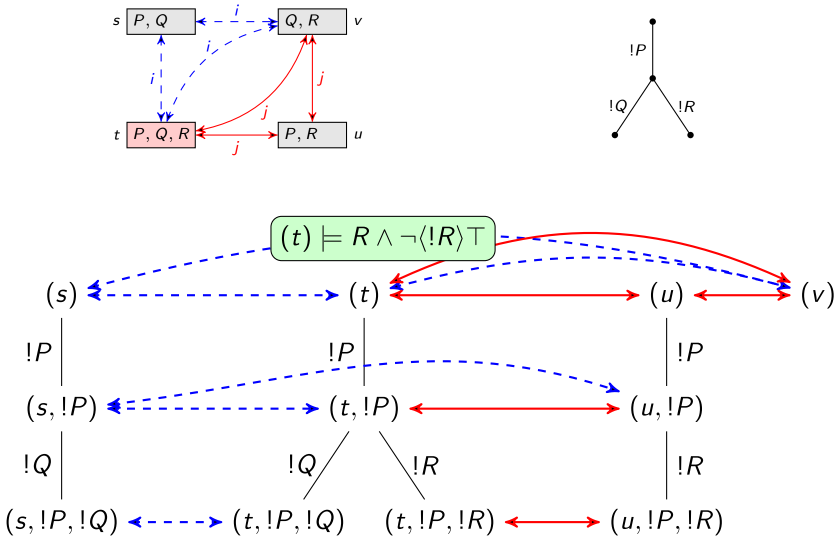
Example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$\mathbb{F}(\mathbf{X}) = \{\text{Forest}(\mathcal{M}, \text{P}) \mid \mathcal{M} \text{ an epistemic model and } \text{P} \in \mathbf{X}\}$.

- Can we characterize the class of ETL models $\mathbb{F}(\mathbf{X})$?
- Can we axiomatize interesting classes of DEL-generated ETL models?

J. van Benthem, J. Gerbrandy, T. Hoshi, EP. *Merging Frameworks for Interaction*. JPL, 2009.

A Characterization Theorem

Let Σ be a finite set of events and suppose $\mathbf{X}^{uni}_{DEL}$ is the class of uniform DEL protocols (with a finiteness condition).

Characterization Theorem A model is in $\mathbb{F}(\mathbf{X}^{uni}_{DEL})$ iff it satisfies propositional stability, synchronicity, perfect recall, local no miracles, and local bisimulation invariance.

Constrained Public Announcement

1. $A \rightarrow \langle A \rangle \top$ vs. $\langle A \rangle \top \rightarrow A$

Constrained Public Announcement

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3. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (A \rightarrow \langle A \rangle P)$

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4. $\langle A \rangle K_i P \leftrightarrow \langle A \rangle \top \wedge K_i (\langle A \rangle \top \rightarrow \langle A \rangle P)$

Questions

1. A public announcement is one specific type of event model, can we axiomatize classes of ETL models generated by other types of event models?

2. Which formal languages are best suited to describe these DEL generated ETL models?

$\mathbb{F}(\mathbf{X}) = \{\mathsf{Forest}(\mathcal{M}, \mathsf{P}) \mid \mathcal{M}$ an epistemic model and $\mathsf{P} \in \mathbf{X}\}$.

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Examples: $\mathbb{F}(\mathsf{PAL})$, $\mathbb{F}(\mathsf{DEL})$, $\mathbb{F}(\mathbf{X}_{PAL})$, $\mathbb{F}(\mathbf{X}_{DEL})$, $\mathbb{F}(\mathbf{X}_{SPriv})$, ...

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Theorems. Sound and complete axiomatizations
(Plaza, Gerbrandy, BMS, van Benthem et al., Hoshi & EP,
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Theorems. Sound and complete axiomatizations (Plaza, Gerbrandy, BMS, van Benthem et al., Hoshi & EP, Balibiani et al., Hoshi, ...)

The logical playground: Decidability of MSO over trees (Rabin); combinations of *PDL* and *S*5; high undecidability just around the corner (Halpern & Vardi, Miller & Moss)

J. van Benthem and EP. *The Tree of Knowledge in Action*. AiML 2006.

Returning to the original questions

How should we represent and reason about the underlying *protocol* (or *plans*) that governs the agents' interactions in a particular social situation?

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3. Logical issues: language design, axiomatization issues

What *is* a Protocol?

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- A protocol is the set of histories satisfying some **property**:
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- A protocol is the set of histories of an extensive game consistent with a (partial) **strategy profile.**

Reasoning about protocols, or plans

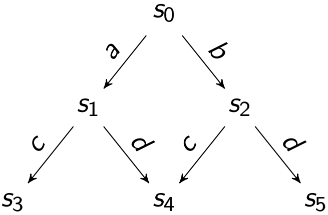
Reasoning about protocols, or plans

When can an agent *agree* to follow a protocol or plan?

What does the agent need to know about the protocol before agreeing to follow it?

EP and Sunil Simon. *Reasoning with Protocols under Imperfect Information*. manuscript.

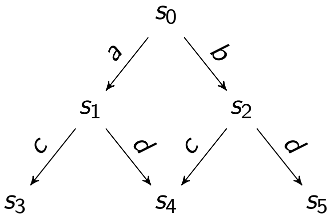
Arena



Protocol or Plan

A **protocol** is a finite tree.

At s_0, the agent agrees to either choose c or choose d:
$(a \cup b); c \cup (a \cup b); d$



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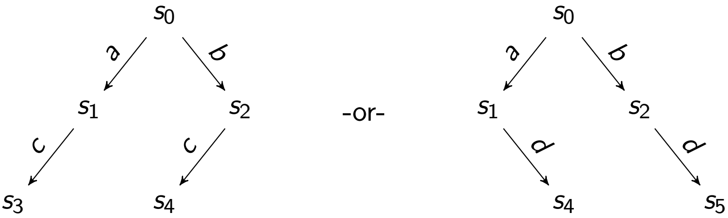

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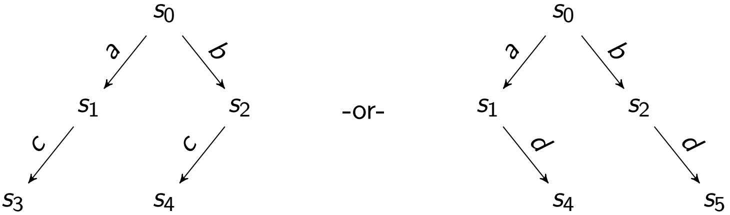

J. van Benthem. *Logical Dynamics of Information and Interaction*. Cambridge University Press, 2010.

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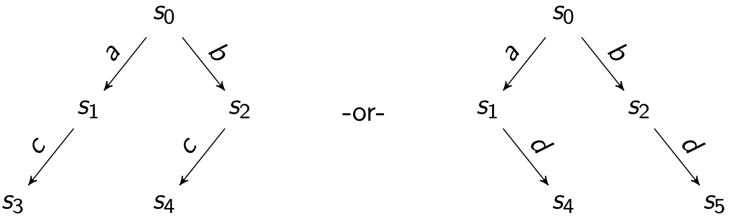
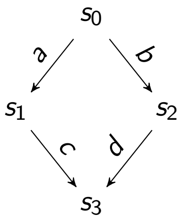
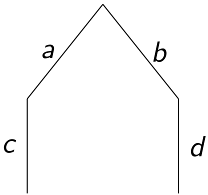


Key idea: of course, PDL action expressions can encode any finite tree, but we want PDL *over trees*

Imperfect Information

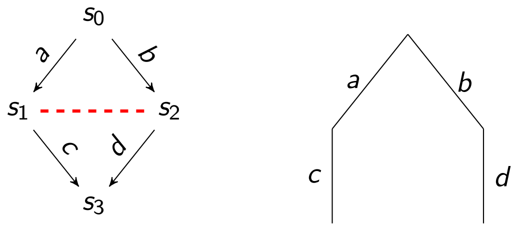
The protocol is **enabled**:





Imperfect Information

The protocol is not **enabled**:



Protocols or Plans

A **protocol** is a finite tree T.

A **complex protocol** is generated by the following grammar:

$$T \mid \pi_1;\pi_2 \mid \pi_1 \cup \pi_2 \mid \pi^*$$

Arena with Imperfect Information

An **arena with imperfect information** is a structure $G^I = (W, \{\Rightarrow_a\}_a \in \Sigma, \rightsquigarrow)$ where $\rightsquigarrow \subseteq W \times W$.

For each position u in an arena, let $\mathcal{I}(u) = \{w \mid u \rightsquigarrow w\}$ be the agent's "*point-of-view*".

For each u, let $\mathcal{A}(u) = \{v \mid \exists a \in \Sigma, w \Rightarrow_a v\}$

Arena with Imperfect Information

- **No Miracles**: for all $a \in \Sigma$ and all $w, v, w', v' \in W$, if $w \rightsquigarrow v$, $w \xrightarrow{a} w'$, and $v \xrightarrow{a} v'$, then $w' \rightsquigarrow v'$.
- **Success**: If $w \rightsquigarrow v$ then $\mathcal{A}(v) \subseteq \mathcal{A}(w)$
- **Awareness**: If $w \rightsquigarrow v$ then $\mathcal{A}(w) \subseteq \mathcal{A}(v)$
- **Certainty of available actions**: If $w \rightsquigarrow v$ and $w \rightsquigarrow v'$ then $\mathcal{A}(v) = \mathcal{A}(v')$

Enabled

A protocol T is **objectively enabled** if T at u in an arena if T can be embedded in the unwinding of u.

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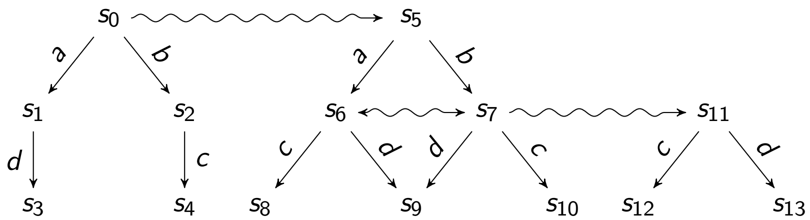
A protocol T is **objectively enabled** if T at u in an arena if T can be embedded in the unwinding of u.

A protocol T is **subjectively enabled** at a position u in an arena with imperfect information if

1. the agent is *certain that* T is enabled (for all $v \in \mathcal{I}(u)$, T is enabled at v), and
2. the agent will be certain that she is in fact following the protocol at *every stage* of the protocol.

Subjectively Enabled: The Idea



Adopting a Protocol

- Committing to a basic protocol T *restricts* the choices available to the agent, but there is a trade-off: it also *increases* the ability of the agent to *guarantee* that certain propositions are true.

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- Committing to a basic protocol T *restricts* the choices available to the agent, but there is a trade-off: it also *increases* the ability of the agent to *guarantee* that certain propositions are true.
- Formally, each basic protocol (which is a finite tree) is associated with a set of states X that the agent can "force" the situation to end up in by making choices consistent with the protocol.

Adopting a Protocol

- There are a number of ways to make precise what it means for an agent to "guarantee" that some proposition is true because she adopts the protocols T.

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- There are a number of ways to make precise what it means for an agent to "guarantee" that some proposition is true because she adopts the protocols T.

- Given a complex protocol π, the agent must first decide both *how* to go about adopting π then make her choices"in the moment" consistent with this plan. (Consider committing to $T_1 \cup T_2$).

Reasoning about Abilities under Imperfect Information

Knowledge/belief:

- $\Box\varphi$: "the agent *has the information* that φ is true"

Abilities:

- $\langle\pi\rangle^{\forall}\alpha$: By adopting the protocol π, α is guaranteed to be true.
- $\langle\pi\rangle^{\exists}\alpha$: By adopting the protocol π, the agent can do something consistent with the protocol that will make α true.

"Epistemized" versions of the above operators:

- $\langle\pi\rangle^{\Box}\alpha$: By *agreeing* to adopt the protocol π, the agent is certain that α is guaranteed to be true.
- $\langle\pi\rangle^{\Diamond}\alpha$: By *agreeing* to adopt the protocol π, the agent is can "knowingly" do something consistent with the protocol that will make α true.

Reasoning about Abilities under Imperfect Information

Knowledge/belief:

- $\mathcal{M}, w \models \Box\varphi$ iff for all v, if $w \rightsquigarrow v$ then $\mathcal{M}, v \models \varphi$.

Abilities:

- $M, u \models \langle\pi\rangle^{\exists}\alpha$ iff $\exists(u, X) \in R_{\pi}^{\exists}$, $\exists w \in X$ such that $M, w \models \alpha$.
- $M, u \models \langle\pi\rangle^{\forall}\alpha$ iff $\exists(u, X) \in R_{\pi}^{\forall}$ such that $\forall w \in X$ we have $M, w \models \alpha$.

"Epistemized" versions of the above operators:

- $M, u \models \langle\pi\rangle^{\Box}\alpha$ iff $\exists(u, X) \in R_{\pi}^{\Box}$ such that $\forall w \in X$ we have $M, w \models \alpha$.
- $M, u \models \langle\pi\rangle^{\Diamond}\alpha$ iff $\exists(u, X) \in R_{\pi}^{\Diamond}$ such that $\forall w \in X$ we have $M, w \models \alpha$.

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Reasoning about Abilities under Imperfect Information

- $R_t^{\mathfrak{F}} = \{(u, X) \mid enabled(T_t, u) \text{ and } frontier(T_u \upharpoonright T_t)) = X\}$ (for $\mathfrak{F} \in \{\exists, \forall\}$).

- $R_t^{\Box} = \{(u, X) \mid s\text{-}enabled(T_t, u) \text{ and } frontier((\mathcal{G}, u) \upharpoonright_s T_t)) = X\}$.

- $R_t^{\Diamond} = \{(u, X) \mid s\text{-}enabled(T_t, u) \text{ and } \exists \rho \in Paths(T_t) \text{ with } \mathfrak{S}(\rho, u) = Z_0 Z_1 \ldots Z_k \text{ and } X = last(Z_k)\}$.

Logics of Information and Abilities

- Combining PDL with epistemic/doxastic logics

R. Schmidt and D. Tishkovsky. *On combinations of propositional dynamic logic and doxastic modal logics*. JOLLI, 2008.

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R. Schmidt and D. Tishkovsky. *On combinations of propositional dynamic logic and doxastic modal logics*. JOLLI, 2008.

- Knowing *how* to execute a plan/win a game

A. Herzig and N. Troquard. *Knowing how to play: uniform choices in logics of agency*. Proceedings of AAMAS 2006, pgs. 209 - 216.

Y. Lesperance, H. Levesque, F. Lin and R. Scherl. *Ability and Knowing How in the Situation Calculus*. Studia Logica 65, pgs. 165 - 186, 2000.

EP and S. Simon. *Reasoning about protocols under imperfect information*. manuscript, 2010.

The Revision Problem

$$State * Input = State'$$

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$$\textit{State} * \textit{Input} = \textit{State}'$$

1. How should we *describe* the "mental state"?
2.
3.
4.

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1. How should we *describe* the "mental state"?
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4. How should we characterize the revision operator?

The Revision Problem

$$\mathcal{B} * \varphi = \mathcal{B}'$$

1. How should we *describe* the "mental state"?
   Propositional formulas
2. What is a *coherent* description?
   A belief state is a consistent and deductively closed set of propositional formulas
3. What are the inputs?
   Consistent propositional formula, other belief sets
4. How should we characterize the revision operator?
   AGM postulates, Dynamic Doxastic Logic

The Revision Problem

$$State * Input = State'$$

What about other aspects of rational agency?

1. How should we *describe* the "mental state"?
   Propositional formulas
2. What is a *coherent* description?
   A belief state is a consistent and deductively closed set of propositional formulas
3. What are the inputs?
   Consistent propositional formula, other belief sets
4. How should we characterize the revision operator?
   AGM postulates, Dynamic Doxastic Logic

Revising Mental Attitudes

✓ Preference change

T. Grüne-Yanoff and S. Ove Hansen (eds.). *Preference Change*. Vol. 42, Theory and Decision Library (2009).

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Today: Intention dynamics

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Important distinctions:

1. (Present-directed) The intention with which someone acts
2. (Present-directed) Intentional action
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Some issues:

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  "Where we are tempted to speak of 'different senses' of a word which is clearly not equivocal, we may infer that we are pretty much in the dark about the character of the concept which it represents"
  - G.E.M. Anscombe, *Intention*, pg. 1

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  pro-attitude (vs. informational attitude), *world-to-mind* direction of fit, *conduct-controlling*

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- Unifying account of *intentions*
- Intention as a *mental state*
- Intentions are (always) directed towards *actions*
  "Although we sometimes report intention as a propositional attitude — 'I intend that *p*' — such reports can always be recast as 'intending to ....' as when I intend to bring about that *p*. By contrast, it is difficult to rephrase such mundane expressions as 'I intend to walk home' in propositional terms"

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"intention is a distinctive practical attitude marked by its pivotal role in planning for the future. Intention involves desire, but even predominant desire is insufficient for intention, since it need not involve a commitment to act: intentions are conduct-controlling pro-attitudes, ones which we are disposed to retain without reconsideration, and which play a significant role as inputs to [means-end] reasoning" (pg. 20)

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Of course, this commitment is *defeasible*...

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What happens in "abnormal" or "surprising" situations? This points to a theory of (rational) *intention/plan revision*...

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2. *Means-ends consistency*: "it is irrational that one intends E, believes that E requires that one intend means M and yet not intend M"
3. *Agglomeration*: "Intending A and Intending B implies Intending (A and B)"

M. Bratman. *Intention, Belief, Practical, Theoretical.* in *Spheres of Reason* (2009).

Intention Dynamics

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Intention Dynamics

1. **Intentional Action Execution**: precise characterization under which an agent's intention *transforms* into an action. (trying, attempting)

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3. Finally, the two problems are *intertwined*: *This is because we must not only maintain that the agent's beliefs and plans are individually consistent, but also that they are jointly coherent.*

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3. the beliefs and intentions together form a coherent picture of what may happen (and how the agent's own actions will play a role in what may happen).

Background: BDI Logics

Stemming from Bratman's planning theory of intention a number of *logics of rational agency* have been developed:

- Cohen and Levesque ▸ Example; Rao and Georgeff (BDI); Meyer and van der Hoek (KARO); and many others.

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Some common features

- Underlying temporal model
- Belief, Desire, Intention, Plans, Actions are defined with corresponding operators in a language

J.-J. Meyer and F. Veltman. *Intelligent Agents and Common Sense Reasoning*. Handbook of Modal Logic, 2007.

A Methodological Issue

What are we formalizing? How will the logical framework be *used*?

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Two Extremes:

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2. Reasoning *about* multiagent *systems*: Three main applications of BDI logics: 1. a specification language for a MAS, 2. a programming language, and 3. verification language.

W. van der Hoek and M. Wooldridge. *Towards a logic of rational agency*. Logic Journal of the IGPL 11 (2), 2003.

Describing the Mental State

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- Intentions are derived from the agents current active plans (trees of practical reasoning rules)

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- Temporal logic, action logic, doxastic logic, combinations, etc., etc.

Focusing the Discussion

Start from an explicit description of *what is being modeled*: eg., a "planner" using a "database" to maintain its current set of beliefs and plans.

Y. Shoham. *Logic of Intention and the Database Perspective*. JPL 2009.

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How should we *describe* the "mental state"?
What type of entries are in the database?

1. Beliefs (about future states, which actions are available plus what the agent might *do*)
2. Current instructions from the planner

Contingent vs. Non-contingent Beliefs

Post-conditions of *intended actions* are justifiably believed *by the mere fact that the agent has committed to bringing them about.*

Contingent vs. Non-contingent Beliefs

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On the other hand, *pre-conditions* may still pose a practical problem yet to be solved.

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[Bratman, pg. 63, my emphasis]

Belief-Intention States: Beliefs

$$\varphi \quad := \quad p_t \mid pre(a)_t \mid post(a)_t \mid Do(a)_t \mid \Box\varphi \mid \varphi \wedge \varphi \mid \neg\varphi$$

$p \in \text{At}$, $a \in \text{Act}$, $t \in \mathbb{Z}^+$

- p_t means p is true at time t
- For every a and time t associate $pre(a)_t$, $post(a)_{t+1}$, which we treat as distinguished propositional variables
- $Do(a)_t$ mean the agent does a at t units from now
- $\Box$ is historic necessity

Belief-Intention States: Beliefs

Beliefs are **consistent and deductively closed** sets of formulas:

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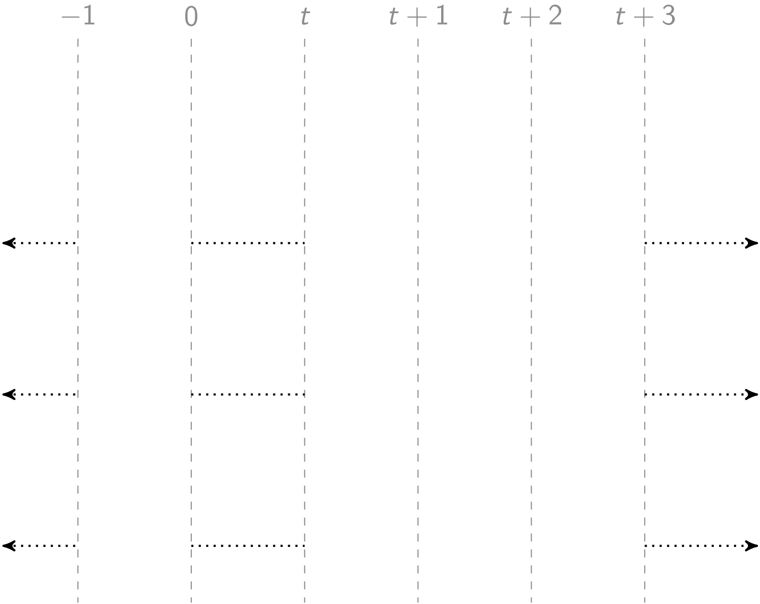
Beliefs are **consistent and deductively closed** sets of formulas:

1. Propositional Tautologies;
2. **S5** axiom schemes for $\Box$: $(\Box(\varphi \rightarrow \psi) \rightarrow \Box\varphi \rightarrow \Box\psi$, $\Box\varphi \rightarrow \varphi$, $\Box\varphi \rightarrow \Box\Box\varphi$,$\Diamond\varphi \rightarrow \Box\Diamond\varphi$, Necessitation);
3. $\bigvee_{a\in \mathsf{Act}} Do(a)_t$;
4. $Do(a)_t \rightarrow \bigwedge_{b\neq a} \neg Do(b)_t$;
5. $Do(a)_t \rightarrow post(a)_{t+1}$;
6. $pre(a)_t \rightarrow \Diamond Do(a)_t$;
7. Modus Ponens

Belief-Intention States: Semantics

$$P = \mathcal{P}(\text{Prop} \cup \{pre(a), post(a) \mid a \in \text{Act}\})$$

Path: $\pi : \mathbb{Z} \rightarrow (P \times \text{Act})$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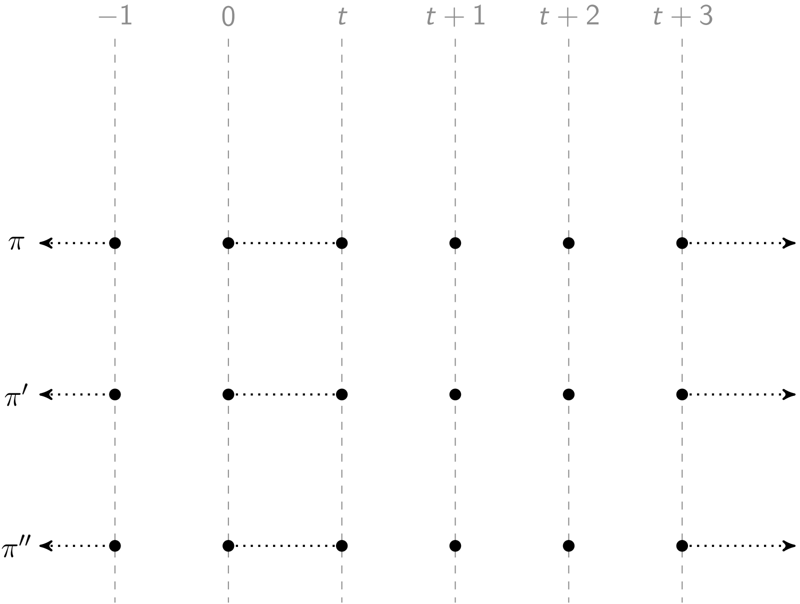
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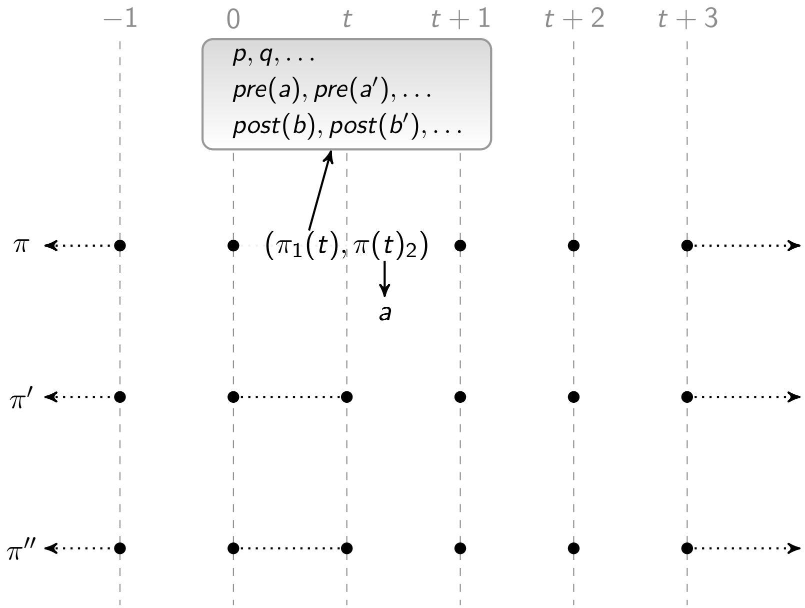
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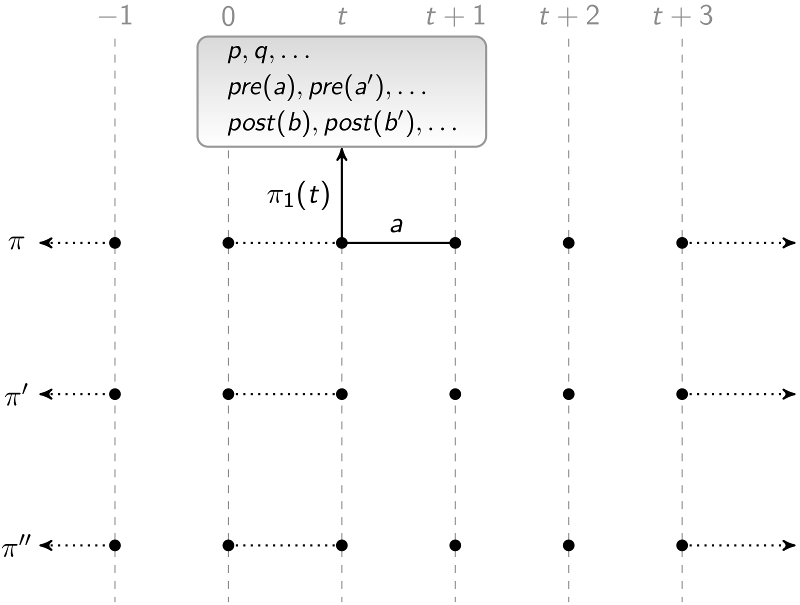
$t+2$
$t+3$
$p, q, \ldots$
$pre(a), pre(a'), \ldots$
$post(b), post(b'), \ldots$
π
$(\pi_1(t), \pi(t)_2)$
a
π'
π''



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t + 2
t + 3
p, q, . . .
pre(a), pre(a′), . . .
post(b), post(b′), . . .
π1(t)
a
π
π′
π′′

Belief-Intention States: Semantics

$P = \mathcal{P}(\mathsf{Prop} \cup \{pre(a), post(a) \mid a \in \mathsf{Act}\})$

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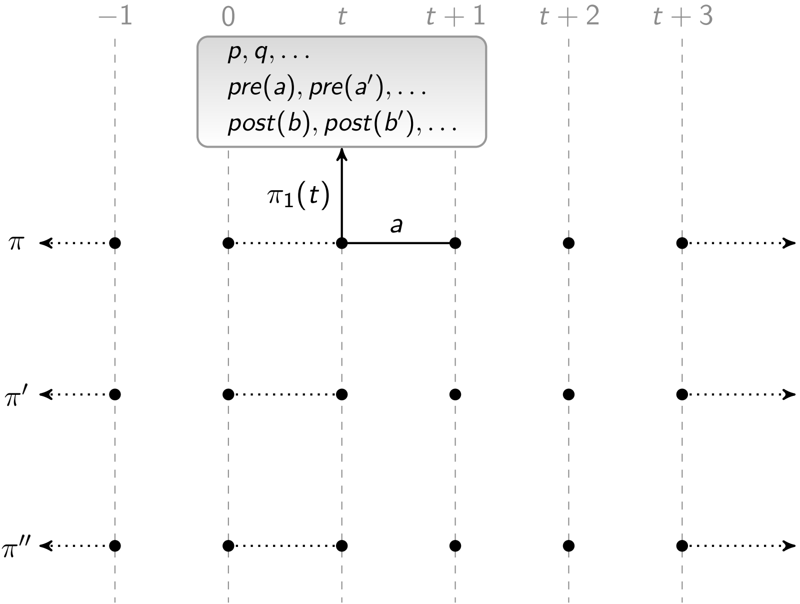
Appropriate path:

- If $\pi(t)_2 = a$ then $post(a) \in \pi(t+1)_1$
- If $pre(a) \in \pi(t)_1$ then there is some $\pi' \sim_t \pi$ such that $\pi'(t)_2 = a$.


−1
0
t
t + 1
t + 2
t + 3
p, q, . . .
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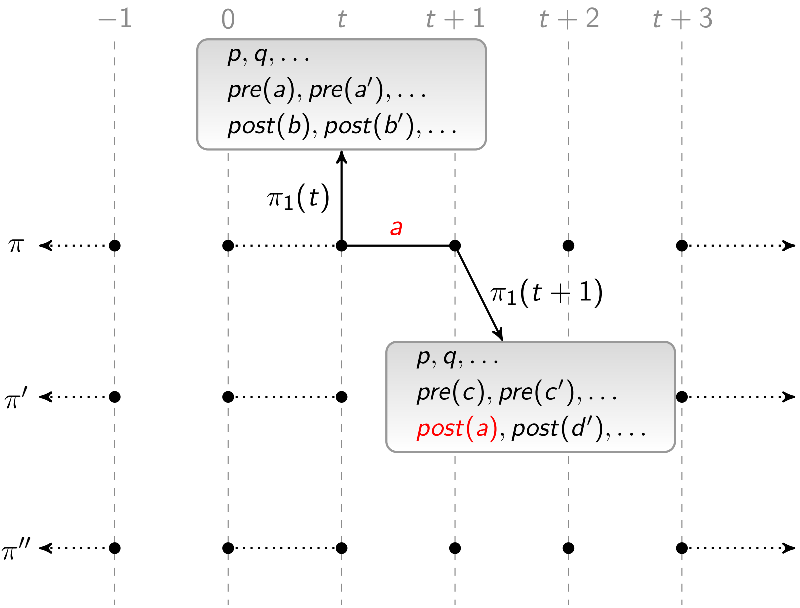

−1
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t
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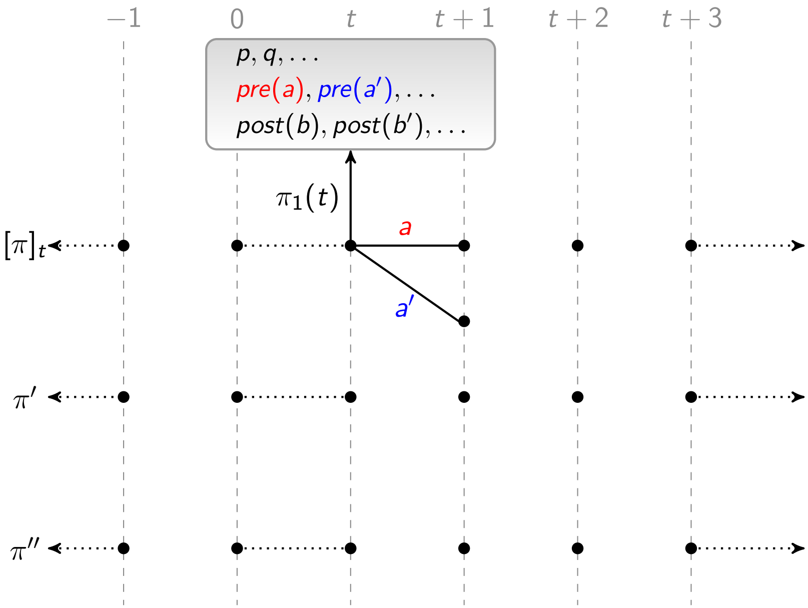
t + 3
p, q, . . .
pre(a), pre(a′), . . .
post(b), post(b′), . . .
π1(t)
a
π
π1(t + 1)
p, q, . . .
pre(c), pre(c′), . . .
post(a), post(d′), . . .
π′
π′′



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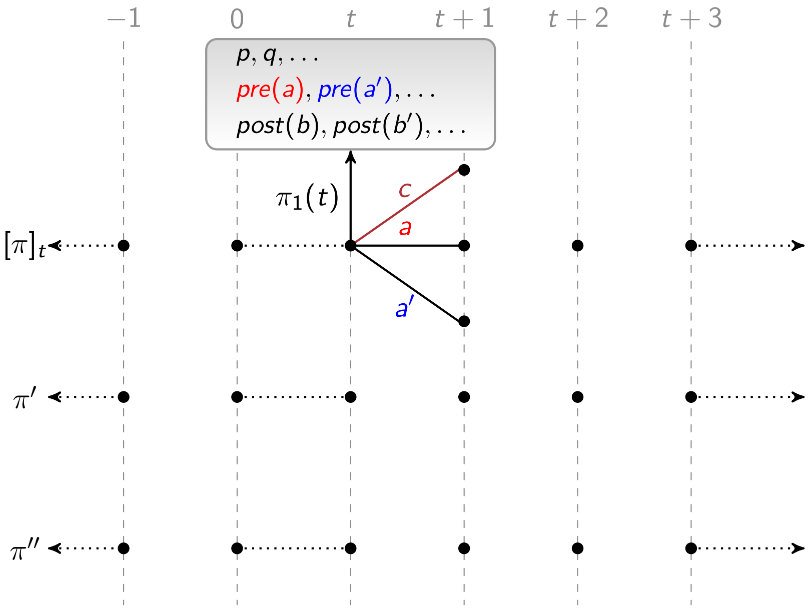
π′
π′′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Belief-Intention States: Semantics

Truth:

- $\pi, t \models \alpha_{t'}$ iff $\alpha \in \pi(t')_1$ for $\alpha = p, pre(a), post(a)$
- $\pi, t \models Do(a)_{t'}$ iff $\pi(t')_2 = a$
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Completeness Theorem. The logic given earlier is sound and complete with respect to the class of all appropriate paths.

Proof. Standard modal reasoning.

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Coherence: $Cohere_I \;:=\; \Diamond \bigwedge_{(a,t)\in I} pre(a)_t$

- (B, I) is *coherent* if $\neg Cohere_I \not\in B$.
- (Π, I) is *coherent* if there is some $\pi \in \Pi$ such that $\pi, 0 \models Cohere_I$.

1. How should we *describe* the "mental state"?
   Sufficiently rich structure/(modal) language

2. What is a *coherent* description?
   What properties/logic characterize a *rational balance*?

3. What are the inputs?

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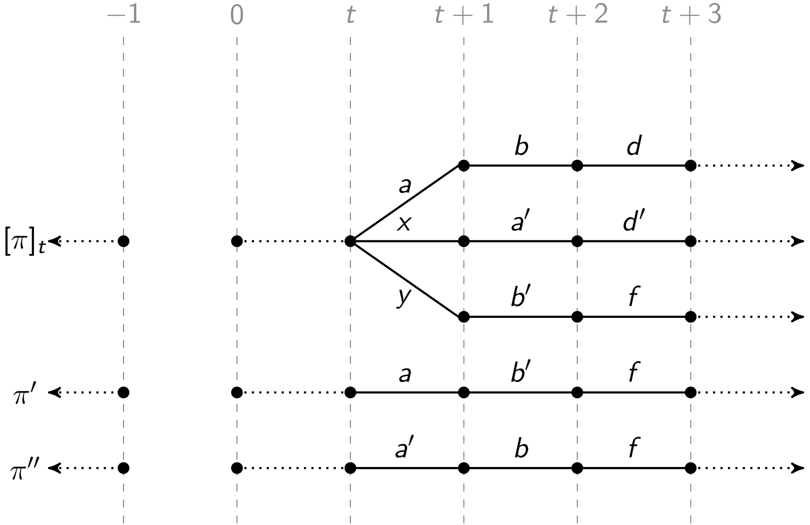

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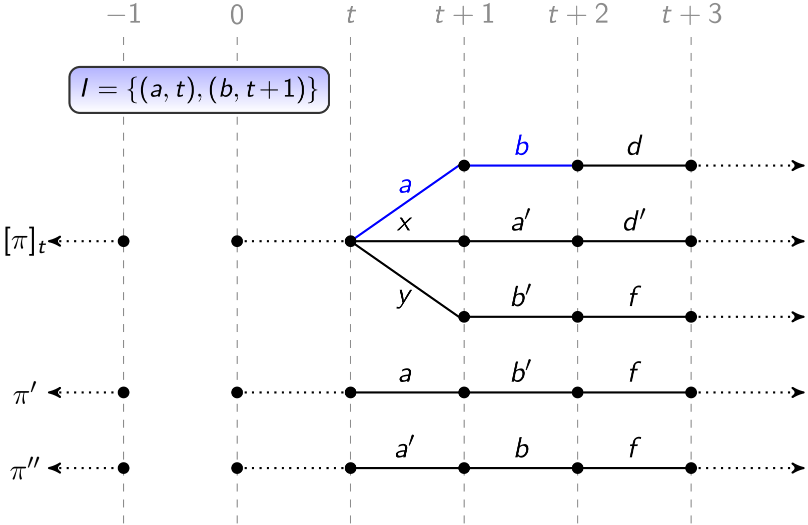

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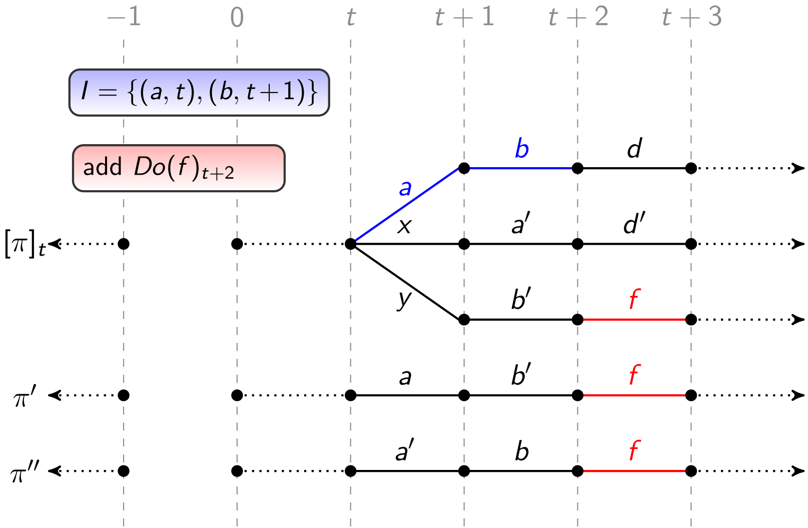

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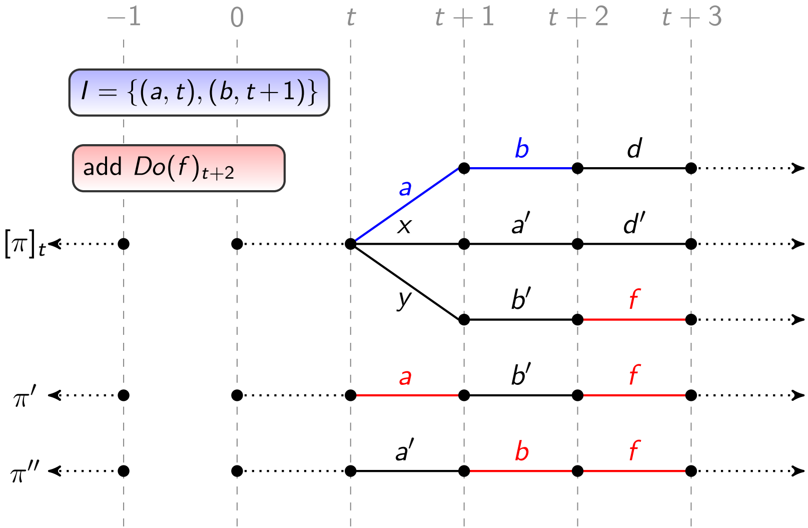

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Intention revision: what is the difference between "add $Do(a)_t$" and "add (a, t) to I"?

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$$(B, I) \quad \star \quad x \quad = \quad (B', I')$$

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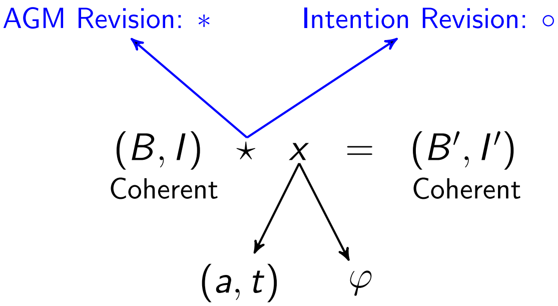
Coherent Coherent

$x \to (a, t)$ $\quad x \to \varphi$

The Revision Problem



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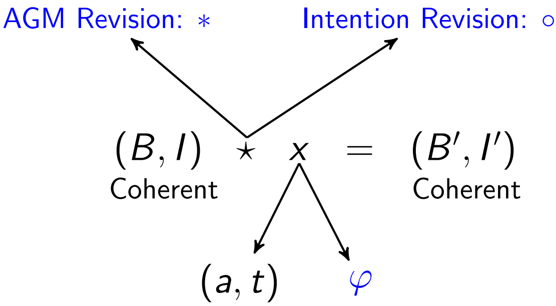

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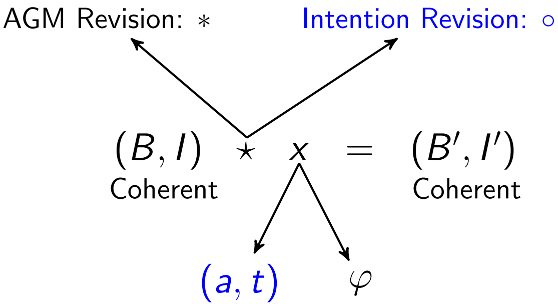

Postulates: Adding an Intention

Suppose (B, I) is coherent and (a, t) and action/time pair.

Intention Revision Operator: $(B, I) \circ (a, t) = (B', I')$ where

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Intention Revision Operator: $(B, I) \circ (a, t) = (B', I')$ where

1. (B', I') is coherent
2. If $(B, \{(a, t)\})$ is coherent then $(a, t) \in I'$
3. If $(B, I \cup \{(a, t)\})$ is coherent then $I \cup \{(a, t)\} \subseteq I'$
4. $B' = B$

Postulates: AGM Revision

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1. $(B', I') = (B', I) \circ \epsilon$ where $\circ$ satisfies the earlier postulates
2. $\varphi \in B'$
3. If $\neg\varphi \notin B$ then $Cl(B \cup \{\varphi\}) = B'$
4. If φ and ψ are *equivalent* and $(B, I) * \psi = (B'', I'')$ then $B' = B''$
5. $B' = Cl(B')$
6. If $\neg\psi \notin B'$ and $(B, I) * \psi = (B'', I'')$ then $Cl(B' \cup \{\psi\}) \subseteq B''$
7. If $(B, I'') * \varphi = (B'', I''')$ then $B' = B''$.

Modeling Revision

$(\Pi, I, \leq, \gamma)$ is a belief intention model where

- Π is an appropriate set of paths
- $\leq$ is a total preorder on Π
- $(min_{\leq}(\Pi), I)$ is coherent
- γ is a **selection function** mapping triples $(\Pi', I', (a,t))$ to J where
  - (Π', J) is coherent
  - If $(\Pi', \{(a,t)\})$ is coherent then $(a,t) \in J$
  - If $(\Pi', I' \cup \{(a,t)\})$ is coherent then $J = I' \cup \{(a,t)\}$
  - $J \subseteq I' \cup \{(a,t)\}$

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Adding a belief: $(\Pi, I, \leq, \gamma) * \varphi = (\Pi, I', \leq', \gamma')$ where $\gamma = \gamma'$, $\leq'$ is the **lexicographic** re-ordering of $\leq$ by φ and $I' = \gamma(min_{\leq'}(\Pi), I, \epsilon)$.

Conclusions

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   Sufficiently rich structure/(modal) language

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- Contingent beliefs: $B^I = Cl(B \cup \{Do(a)_t \mid (a,t) \in I\})$. We can read off postulates.

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- Other mental attitudes...

Logics of Rational Agency

- Special issue of JOLLI: Temporal Logics of Agency (eds. J. van Benthem and EP)
- Special Issue of Synthese: Knowledge, Rationality and Interaction. *Logic and Intelligent Interaction*, Volume 169, Number 2 / July, 2009
  (eds. T. Agotnes, J. van Benthem and EP)
- New subarea of Stanford Encyclopedia of Philosophy on logic and rational agency
  (eds. J. van Benthem, EP, and O. Roy)

Thank You!

Dynamic Operator

[Recall a BDI state: $\Gamma_0 = \langle \mathcal{B}_0, \mathcal{D}_0, \langle \mathcal{I}_0, \mathcal{A}_0 \rangle, \P_0 \rangle$
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3. $\mathcal{I}_1 = \mathit{Int}(\mathcal{A}_1)$

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- $(D\varphi \wedge I\varphi \wedge \neg[\Omega]B\varphi \wedge \neg[\Omega]B\neg\varphi) \rightarrow [\Omega]I\varphi$

Back to conclusions

C & L Logic of Intention

1. Intentions normally pose problems for the agent; the agent needs to determine a way to achieve them.
2. Intentions provide a "screen of admissibility" for adopting other intentions.
3. Agents "track" the success of their attempts to achieve their intentions.
4. If an agent intends to achieve p, then
   - 4.1 The agent believes p is possible
   - 4.2 The agent does not believe he will not bring abut p
   - 4.3 Under certain conditions, the agent believes he will bring about p
   - 4.4 Agents need not intend all the expected side-effects of their intentions.

C & L Logic of Intention

$$(\text{PGOAL}_i p) := (\text{GOAL}_i(\text{LATER}p)) \wedge$$
$$(\text{BEL}_i \neg p) \wedge [\text{BEFORE}((\text{BEL}_i p) \vee (\text{BEL}_i \Box \neg p)) \neg (\text{GOAL}_i(\text{LATER}p))]$$

$$(\text{INTEND}_i a) := (\text{PGOAL}_i[\text{DONE}_i(\text{BEL}_i(\text{HAPPENS}a))?; a])$$

Back