From Relational to Neighborhood Models

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Background

- Propositional modal language.
- Relational frames/models for modal logic.
- Definition of truth of modal formulas at states in a relational model.

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- Relational frames/models for modal logic.
- Definition of truth of modal formulas at states in a relational model.

See Appendix A: Relational Semantics for Modal Logic.

The Basic Modal Language: $\mathcal{L}$

$$p \mid \neg\varphi \mid \varphi \wedge \psi \mid \Box\varphi \mid \Diamond\varphi$$

where p is an atomic proposition (Let At be the set of atomic propositions)

Relational Structures

Relational (Kripke) Frame: $\langle W, R\rangle$

- $W \neq \emptyset$
- $R \subseteq W \times W$

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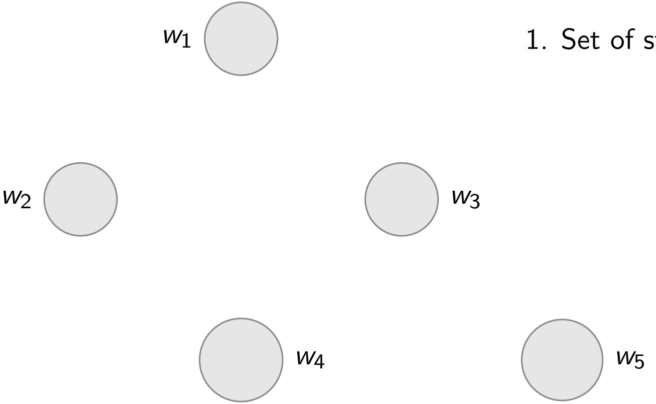
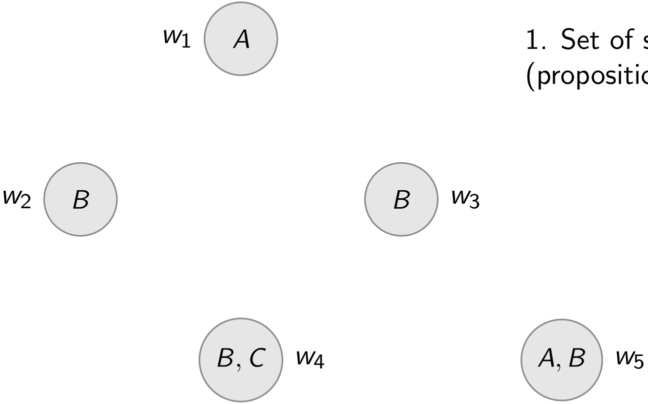
Relational (Kripke) Model: $\langle W, R, V \rangle$

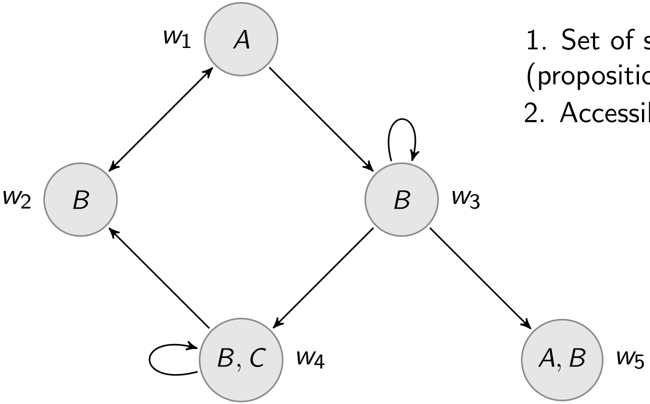
- $\langle W, R \rangle$ is a frame
- $V : \mathsf{At} \to \wp(W)$



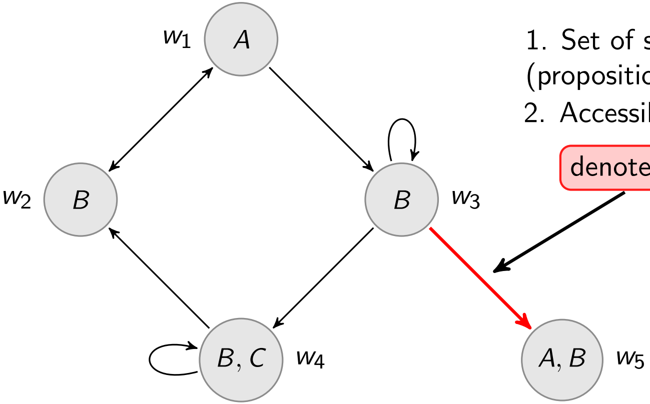
1. Set of states



1. Set of states (propositional valuations)



1. Set of states (propositional valuations)
2. Accessibility relation



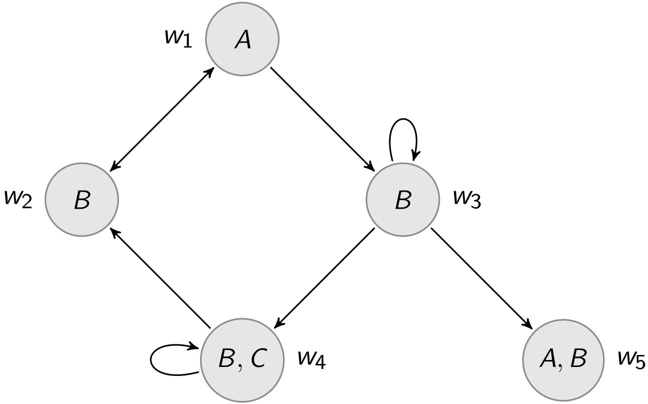
1. Set of states (propositional valuations)
2. Accessibility relation

denoted $w_3 R w_5$

Truth: $\mathcal{M}, w \models \varphi$

1. $\mathcal{M}, w \models p$ iff $w \in V(p)$
2. $\mathcal{M}, w \models \neg\varphi$ iff $\mathcal{M}, w \not\models \varphi$
3. $\mathcal{M}, w \models \varphi \wedge \psi$ iff $\mathcal{M}, w \models \varphi$ and $\mathcal{M}, w \models \psi$
4. $\mathcal{M}, w \models \Box\varphi$ iff for each $v \in W$, if $w\, R\, v$ then $\mathcal{M}, v \models \varphi$
5. $\mathcal{M}, w \models \Diamond\varphi$ iff there is a $v \in W$ such that $w\, R\, v$ and $\mathcal{M}, v \models \varphi$

Example



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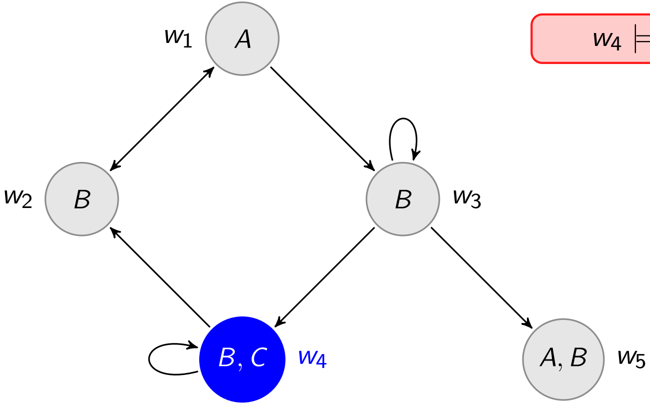
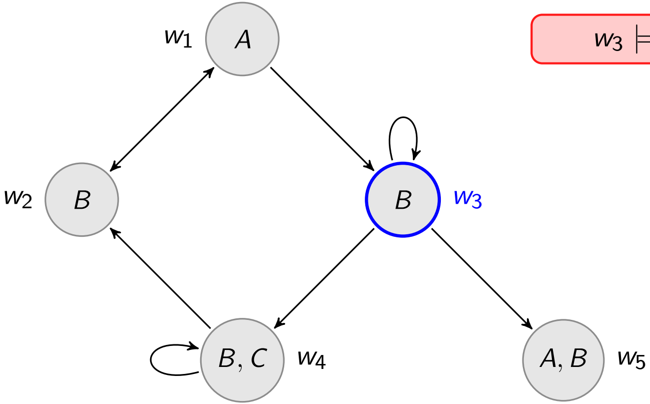


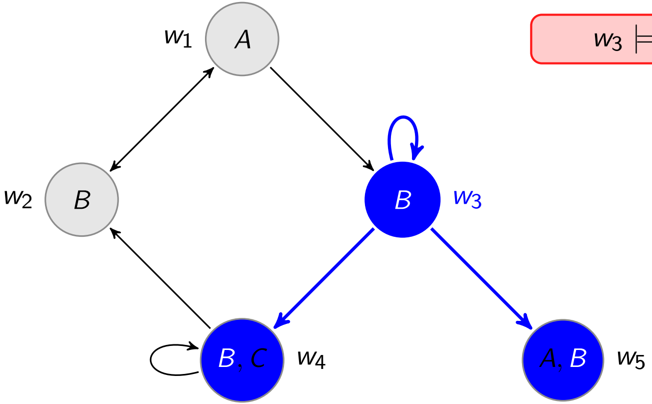
$w_4 \models B \wedge C$

Example



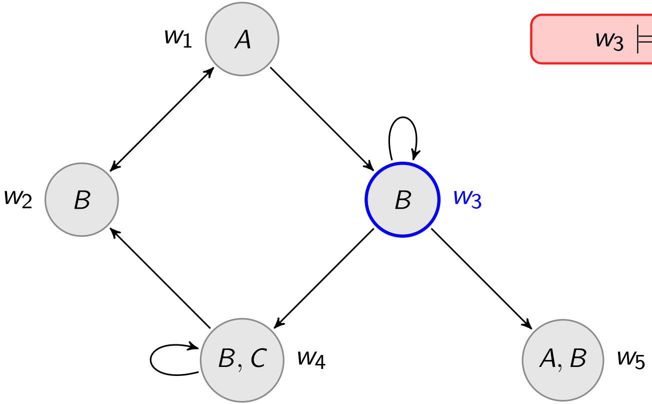
$w_3 \models \Box B$

Example



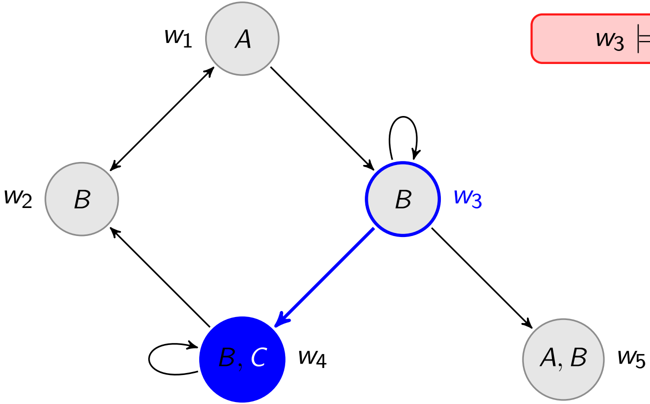
$w_3 \models \Box B$

Example



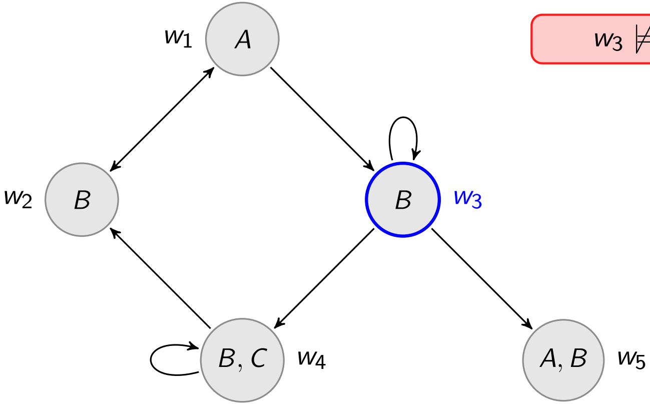
$w_3 \models \Diamond C$

Example



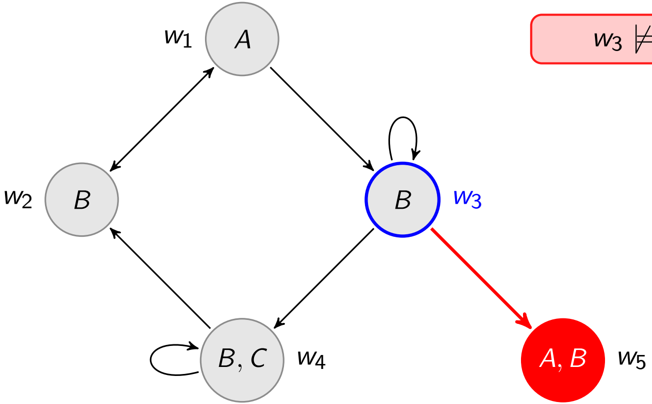
$w_3 \models \Diamond C$

Example

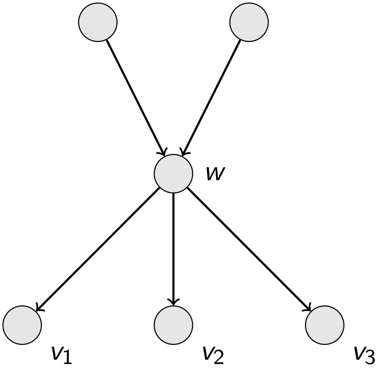


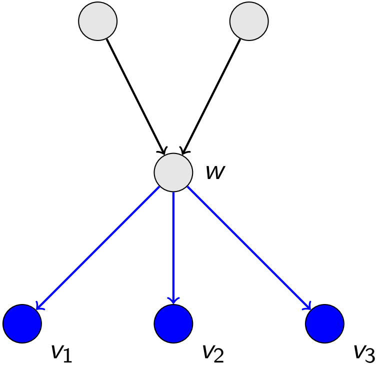
$w_3 \not\models \Box C$

Example

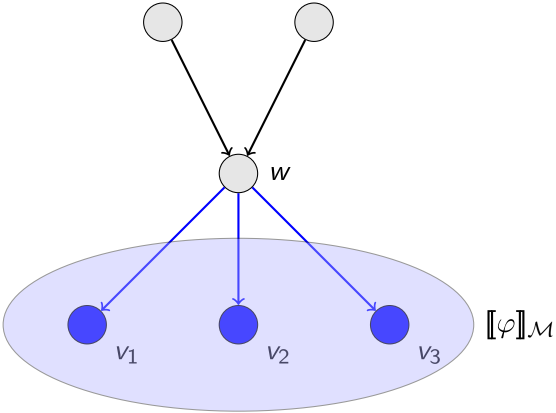


$w_3 \not\models \Box C$


w
v_1
v_2
v_3

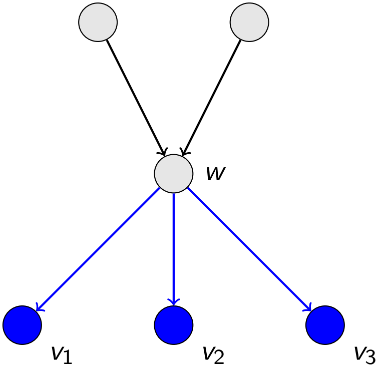


$R(w) = \{v_1, v_2, v_3\}$ is the "neighborhood" of w.



$$\mathcal{M}, w \models \Box\varphi \text{ iff } R(w) \subseteq [\![\varphi]\!]_{\mathcal{M}}$$

...**the** neighborhood of w is
contained in the truth-set of φ



$$\mathcal{M}, w \models \boxplus\varphi \text{ iff } R(w) = [\![\varphi]\!]_{\mathcal{M}}$$

...**the** neighborhood of w **is** the truth-set of φ

$w \models \Box\varphi$ if the truth set of φ is a neighborhood of w

$$w \models \Box\varphi \quad \text{if the truth set of } \varphi \text{ is a neighborhood of } w$$

What does it mean to be a neighborhood?

$w \models \Box\varphi$ if the truth set of φ is a neighborhood of w

neighborhood in some topology.

J. McKinsey and A. Tarski. *The Algebra of Topology*. 1944.

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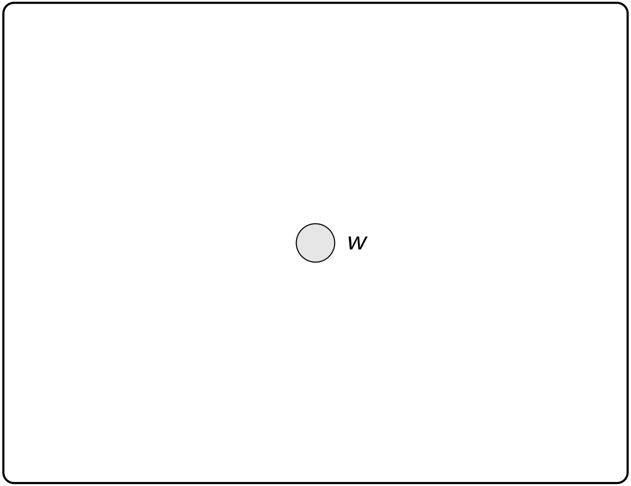
an element of some distinguished collection of sets

D. Scott. *Advice on Modal Logic*. 1970.

R. Montague. *Pragmatics*. 1968.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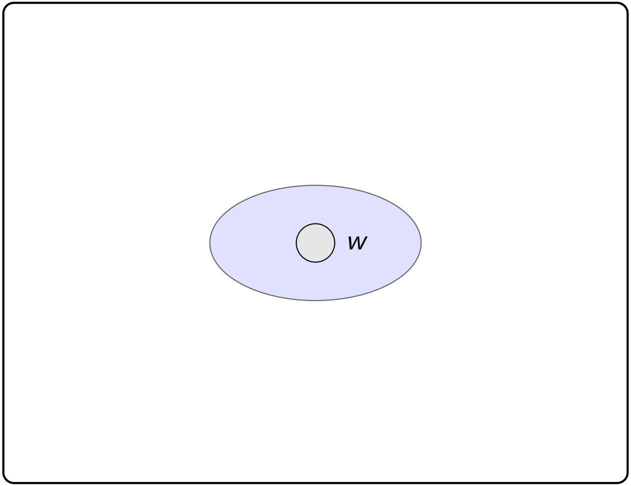
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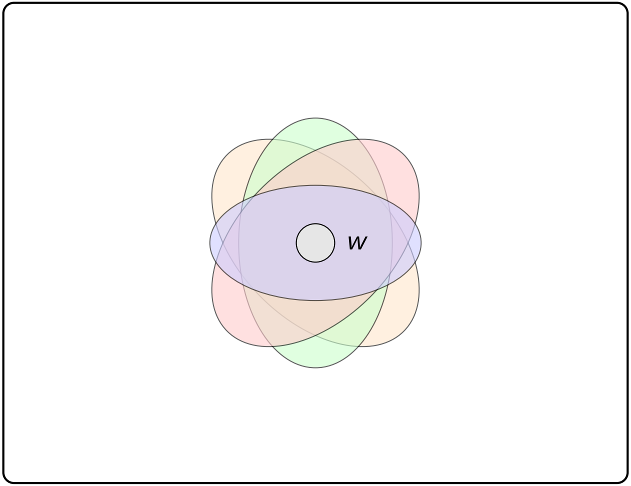
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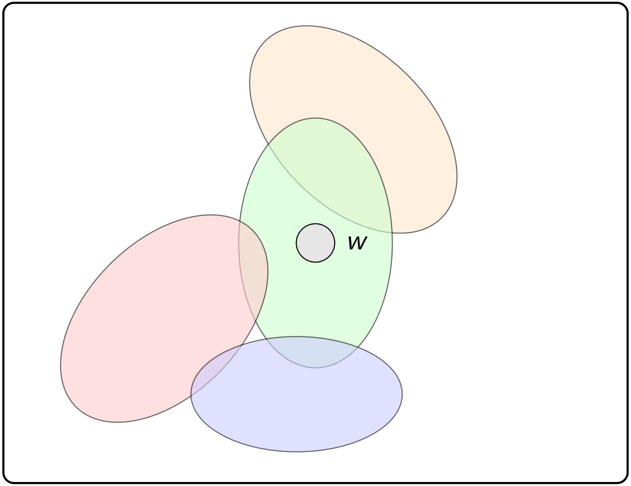
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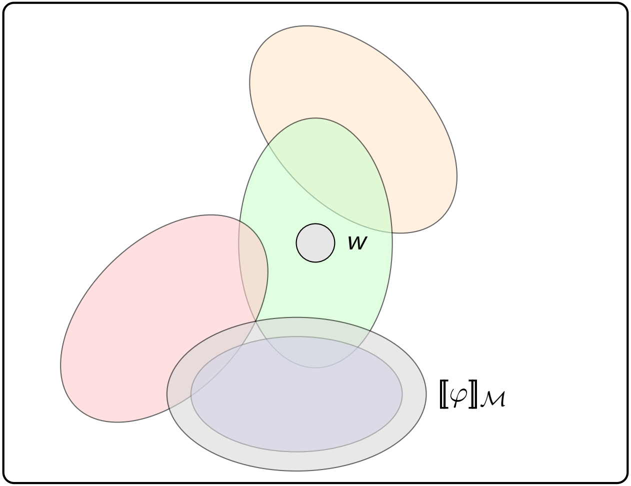


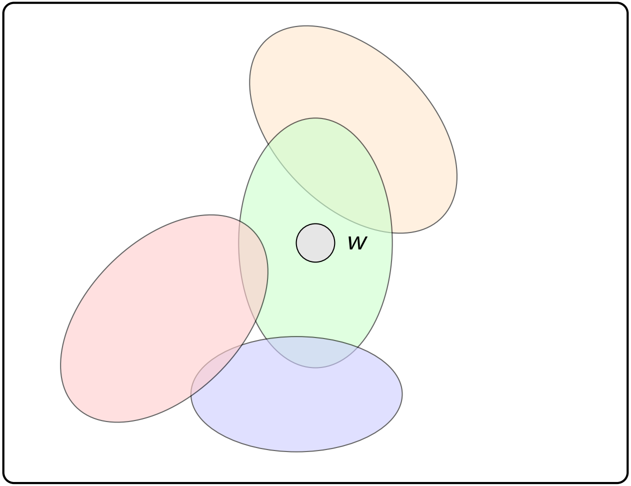
$\mathcal{M}, w \models \Box\varphi$ iff **there is** a neighborhood of w **contained in** $[\![\varphi]\!]_{\mathcal{M}}$

Relational model: $\langle W, R, V \rangle$ where $R : W \to \wp(W)$

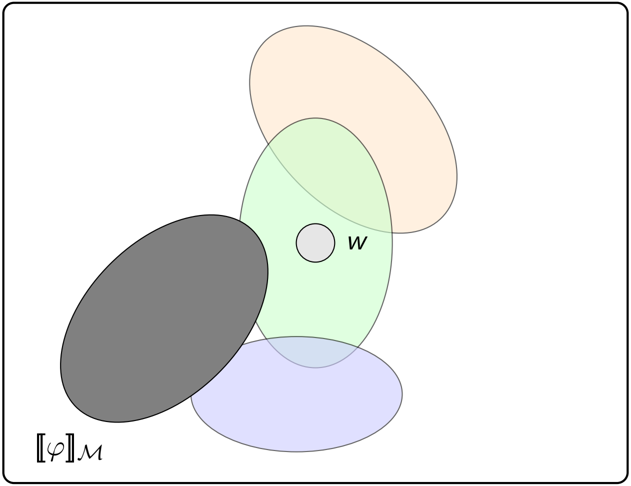
$w \models \Box\varphi$ iff $R(w) \subseteq [\![\varphi]\!]$

Neighborhood model: $\langle W, N, V \rangle$ where $N : W \to \wp(\wp(W))$

$w \models \Box\varphi$ iff there is a $X \in N(w)$ such that $X \subseteq [\![\varphi]\!]$



$\mathcal{M}, w \models \Box\varphi$ iff $[\![\varphi]\!]_{\mathcal{M}}$ **is** a neighborhood of w



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$w \models \Box\varphi$ iff $[\![\varphi]\!] \in N(w)$

$w \models \langle\ ]\varphi$ iff there is a $X \in N(w)$ such that $X \subseteq [\![\varphi]\!]$

See *Neighborhood Semantics for Modal Logic* by Eric Pacuit for more information...

`pacuit.org/modal/neighborhoods`