Models of Strategic Reasoning
Lecture 5

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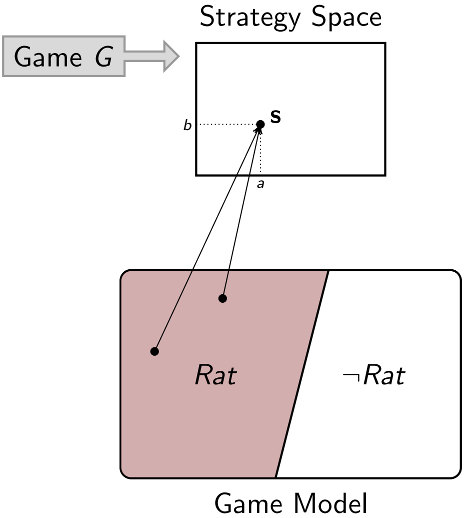
Game Plan

- ✓ Introduction, Motivation and Background
- ✓ The Dynamics of Rational Deliberation
- ✓ Reasoning to a Solution: Common Modes of Reasoning in Games

Lecture 4: Reasoning to a Model: Iterated Belief Change as Deliberation

Lecture 5: Reasoning in Specific Games

Informational Context of a Game



Reasoning *to* a context

"It is important to understand that we have two forms of irrationality in this paper...For us, a player is rational if he optimizes and also rules nothing out. So irrationality might mean not optimizing. But it can also mean optimizing while not considering everything possible."

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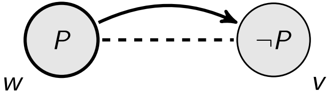
A player can be rationally criticized for

1. not choosing what is *best* or what is *rationally permissible*, *given one's information*.
2. not reasoning to a "proper" informational context.

Recall...



Epistemic-Plausibility Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\preceq_i\}_{i\in\mathcal{A}}, V\rangle$

- $w \preceq_i v$ means v is at least as plausibility as w for agent i.

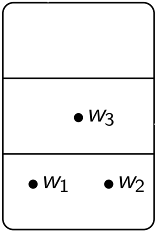
Language: $\varphi := p \mid \neg\varphi \mid \varphi\wedge\psi \mid K_i\varphi \mid B^\varphi\psi \mid [\preceq_i]\varphi$

Truth:

- $[\![\varphi]\!]_{\mathcal{M}} = \{w \mid \mathcal{M}, w \models \varphi\}$
- $\mathcal{M}, w \models B_i^\varphi\psi$ iff for all $v \in Min_{\preceq_i}([\![\varphi]\!]_{\mathcal{M}} \cap [w]_i)$, $\mathcal{M}, v \models \psi$
- $\mathcal{M}, w \models [\preceq_i]\varphi$ iff for all $v \in W$, if $v \preceq_i w$ then $\mathcal{M}, v \models \varphi$

Recall...

- $w_1 \sim w_2 \sim w_3$



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- $w_1 \sim w_2 \sim w_3$
- $w_1 \preceq w_2$ and $w_2 \preceq w_1$ (w_1 and w_2 are equi-plausbile)
- $w_1 \prec w_3$ ($w_1 \preceq w_3$ and $w_3 \not\preceq w_1$)
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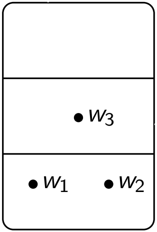

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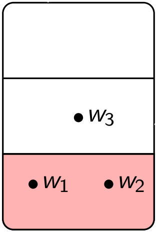
- $\{w_1, w_2\} \subseteq Min_{\preceq}([w_i])$



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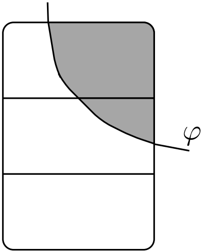
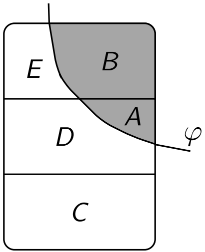


Incorporate the new information φ

Recall...



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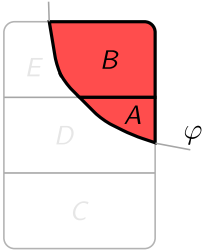

Public Announcement: Information from an infallible source

$(!\varphi)$: $A \prec_i B$

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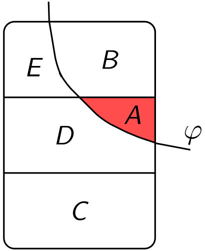

Public Announcement: Information from an infallible source
$(!\varphi)$: $A \prec_i B$

Conservative Upgrade: Information from a trusted source
$(\uparrow\varphi)$: $A \prec_i C \prec_i D \prec_i B \cup E$

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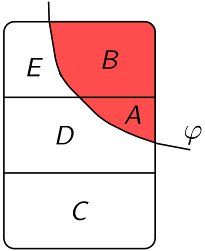

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Radical Upgrade: Information from a strongly trusted source
$(\Uparrow\varphi)$: $A \prec_i B \prec_i C \prec_i D \prec_i E$

Key Idea

Informational contexts of a game arise as fixed points of iterated "rationality announcements".

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K. Apt and J. Zvesper. *Public announcements in strategic games with arbitrary strategy sets*. Proceedings of LOFT 2010 (2010).

J. van Benthem, and A. Gheerbrant. *Game solution, epistemic dynamics and fixed-point logics*. Fund. Inform. 100 (2010), 1-23.

Iterated belief revision as a model of deliberation

Reasoning *about* (strategic) games

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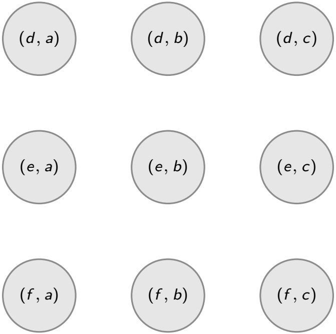
There is Kripke structure "built in" a strategic game.

$W = \{\sigma \mid \sigma \text{ is a strategy profile: } \sigma \in \Pi_{i \in N} S_i\}$

	a	b	c
d	(2,3)	(2,2)	(1,1)
e	(0,2)	(4,0)	(1,0)
f	(0,1)	(1,4)	(2,0)



Reasoning *about* (strategic) games

$\sigma \sim_i \sigma'$ iff $\sigma_i = \sigma'_i$: this epistemic relation represents player i's "view of the game" at the *ex interim* stage where i's choice is fixed but the choices of the other players' are unknown

	a	b	c
d	(2,3)	(2,2)	(1,1)
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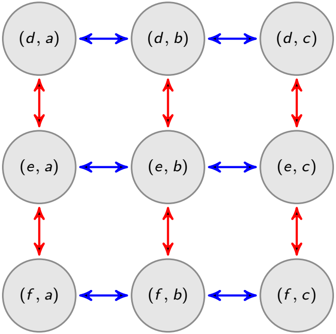
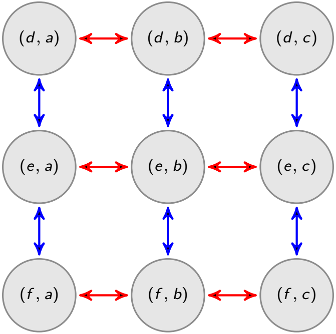


Reasoning *about* (strategic) games

$\sigma \approx_i \sigma'$ iff $\sigma_{-i} = \sigma_{-i}$: this relation of "action freedom" gives the alternative choices for player i when the other players' choices are fixed.

	a	b	c
d	(2,3)	(2,2)	(1,1)
e	(0,2)	(4,0)	(1,0)
f	(0,1)	(1,4)	(2,0)



Reasoning *about* (strategic) games

$\sigma \succeq_i \sigma'$ iff player i prefers the outcome σ at least as much as outcome σ'

	a	b	c
d	(2,3)	(2,2)	(1,1)
e	(0,2)	(4,0)	(1,0)
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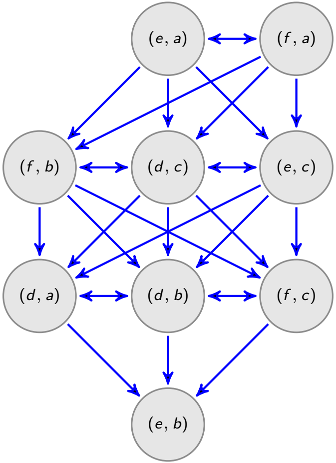

Reasoning *about* (strategic) games

$$\mathcal{M} = \langle W, \{\sim_i\}_{i\in N}, \{\approx_i\}_{i\in N}, \{\succeq_i\}_{i\in N}\rangle$$

- $\sigma \models [\sim_i]\varphi$ iff for all σ', if $\sigma \sim_i \sigma'$ then $\sigma' \models \varphi$.
- $\sigma \models [\approx_i]\varphi$ iff for all σ', if $\sigma \approx_i \sigma'$ then $\sigma' \models \varphi$.
- $\sigma \models \langle\succeq_i\rangle\varphi$ iff there exists σ' such that $\sigma' \succeq_i \sigma$ and $\sigma' \models \varphi$.
- $\sigma \models \langle\succ_i\rangle\varphi$ iff there is a σ' with $\sigma' \succeq_i \sigma$, $\sigma \not\succeq_i \sigma'$, and $\sigma' \models \varphi$

Rationality Announcements

	a	b	c
d	(2,3)	(2,2)	(1,1)
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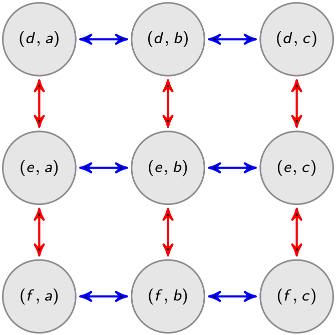

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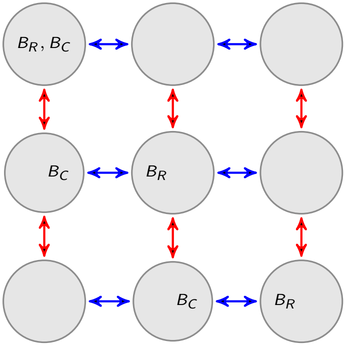


Rationality Announcements: Theorem

Weak Rationality: $w \models WR_j$ means $\bigwedge_{a \neq w(j)}$ 'j *thinks* that j's current action is at least as good for j as a.', where the a's run over the *current* model.

Theorem The following are equivalent for all states s in a full game model

1. s survives iterated removal of strongly dominated strategies
2. repeated successive **public announcements** of WR for the players stabilizes at a submodel whose domain contains s.

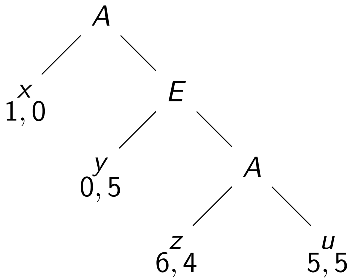
J. van Benthem. *Rational dynamics and epistemic logic in games*. International Game Theory Review 9, 1 (2007), 13-45.

Dynamics for the tree

Where do the models satisfying common knowledge/belief of rationality come from?

J. van Benthem and A. Gheerbrant. *Game solution, epistemic dynamics and fixed-point logics*. Fund. Inform., 100 (2010) 1–23..

Dynamics for the tree



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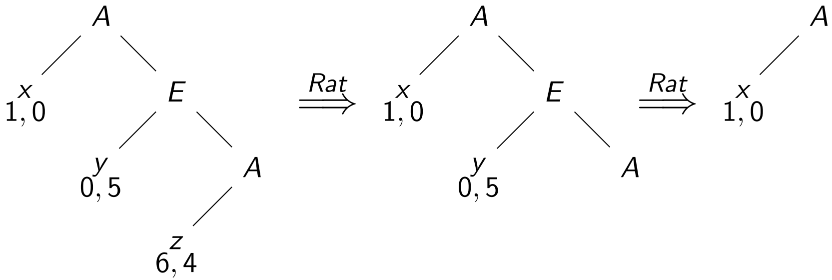

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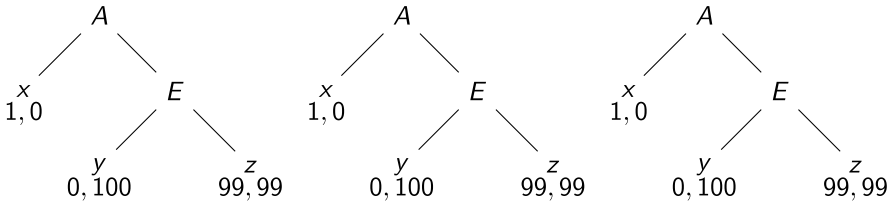

$$x \quad y \quad z \quad \overset{\Uparrow rat}{\Longrightarrow} \quad x \quad y > z \quad \overset{\Uparrow rat}{\Longrightarrow} \quad x > y > z$$

The Dynamics of Rational Play

A. Baltag, S. Smets and J. Zvesper. *Keep 'hoping' for rationality: a solution to the backward induction paradox*. Synthese, 169, pgs. 301 - 333, 2009.

Hard vs. Soft Information in a Game

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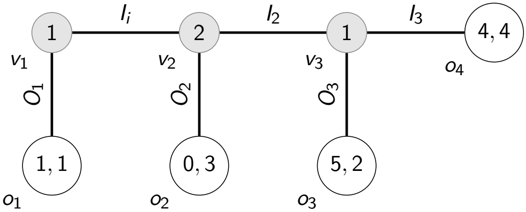
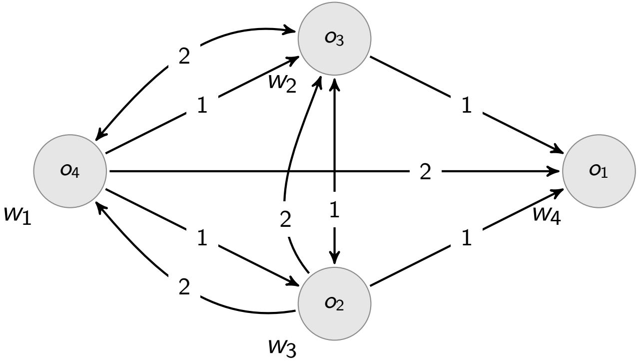

1
2
1
4, 4
l_i
l_2
l_3
v_1
v_2
v_3
o_4
O_1
O_2
O_3
1, 1
0, 3
5, 2
o_1
o_2
o_3


o_3
o_4
o_1
o_2
w_1
w_2
w_3
w_4
2
1
1
2
1
2
1
1
1
2

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$$\mathsf{v} := \bigvee_{v \rightsquigarrow o} \mathsf{o}$$

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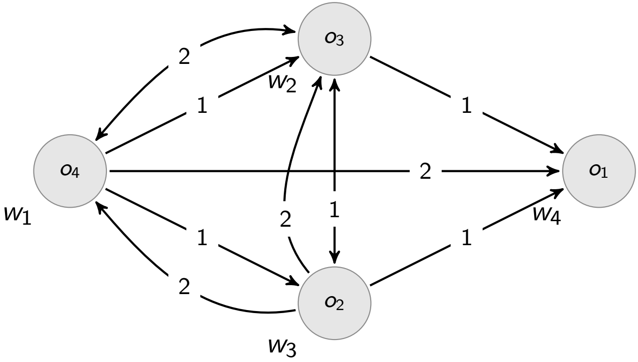
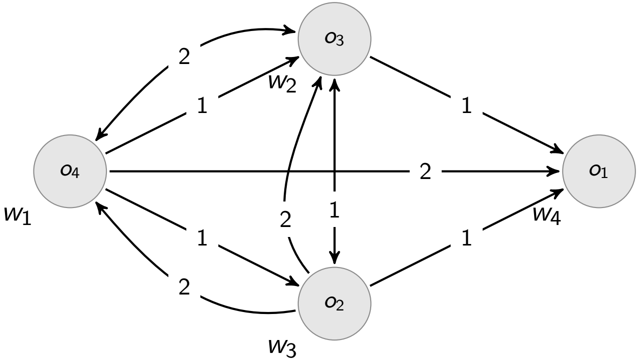
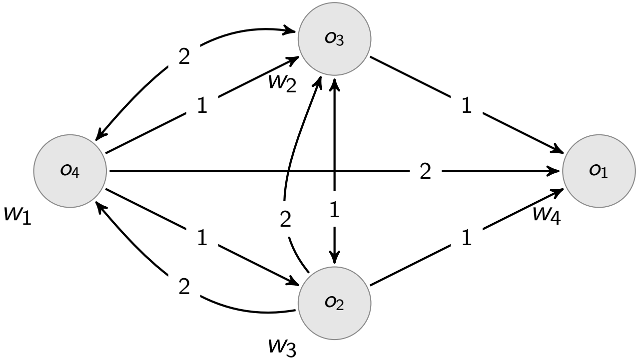
Open future: none of the players have "hard information" that an outcome is ruled out



o_3
o_4
o_1
o_2
w_1
w_2
w_3
w_4
2
1
1
2
2
1
1
1
2



Player 1 is committed to the BI strategy is encoded in the conditional beliefs of the player: both $B_1^{v_1}o_1$ and $B_1^{v_3}o_3$ are true in the previous model.



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For player 2, $B_2^{v_2}(o_3 \vee o_4)$ is true in the above model, which implies player 2 plans on choosing action l_2 at node v_2.

The players' belief change as they learn (irrevocably) which of the nodes in the game are reached:

$$\mathcal{M} = \mathcal{M}^{!v_1}; \mathcal{M}^{!v_2}; \mathcal{M}^{!v_3}; \mathcal{M}^{!o_4}$$

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The assumption that the players are "incurably optimistic" is represented as follows: no matter what true formula is publicly announced (i.e., no matter how the game proceeds), there is common belief that the players will make a rational choice (when it is their turn to move).

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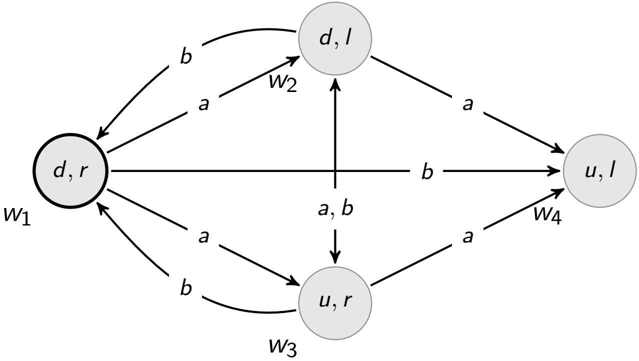
$\mathcal{M}, w \models [\ !\ ]\varphi$ provided for all formulas ψ if $\mathcal{M}, w \models \psi$ then $\mathcal{M}, w \models [!\psi]\varphi$.

Theorem (Baltag, Smets and Zvesper). Common knowledge of the game structure, of open future and *common stable belief* in dynamic rationality implies common belief in the backward induction outcome.

$$Ck(Struct_G \wedge F_G \wedge [\,!\,]CbRat) \to Cb(BI_G)$$

Epistemic-plausibility models for strategic games

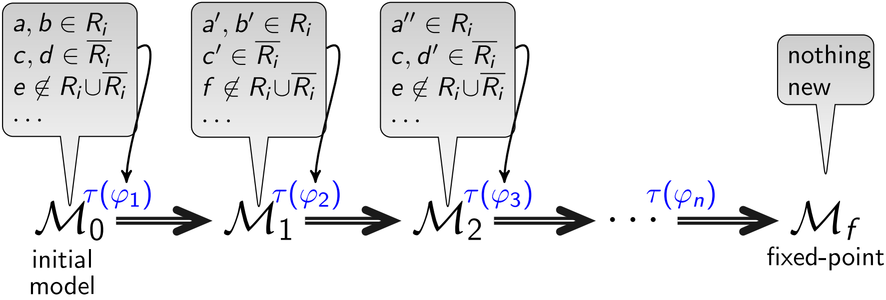
	l	r
u	$3,3$	$0,0$
d	$0,0$	$1,1$



EP and O. Roy. *A Dynamic Analysis of Interactive Rationality*. Proceedings of LORI-III, 2011.

$$\underset{\text{initial model}}{\mathcal{M}_0} \overset{\tau(\varphi_1)}{\Longrightarrow} \mathcal{M}_1 \overset{\tau(\varphi_2)}{\Longrightarrow} \mathcal{M}_2 \overset{\tau(\varphi_3)}{\Longrightarrow} \cdots \overset{\tau(\varphi_n)}{\Longrightarrow} \underset{\text{fixed-point}}{\mathcal{M}_f}$$

Where do the φ_k come from?



Where do the φ_k come from? from the players' practical reasoning (i.e., their *categorization* of their feasible moves)

Iterated Admissibility

Ann (rows) / Bob (columns)

	L	R
T	1,1	1,0
B	1,0	0,1

Iterated Admissibility

		Bob	
		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

T weakly dominates B

Iterated Admissibility

		Bob L	Bob R
Ann	T	1,1	1,0
Ann	B	1,0	0,1

Then L strictly dominates R.

Iterated Admissibility

Bob

Ann		L	R
	T	1,1	1,0
	B	1,0	0,1

The IA set

Iterated Admissibility

Bob

		L	R
Ann	T	1,1	1,0
	B	1,0	0,1

But, now what is the reason for not playing B?

A Dynamic Analysis of Iterated Admissibility

	L	R
u	1,1	1,0
d	1,0	0,1

A Dynamic Analysis of Iterated Admissibility

	L	R
u	1,1	1,0
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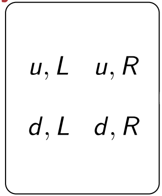

$\mathcal{M}_0$

A Dynamic Analysis of Iterated Admissibility

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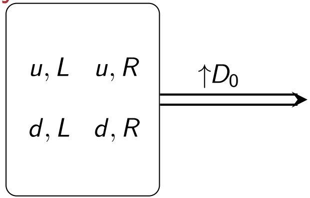

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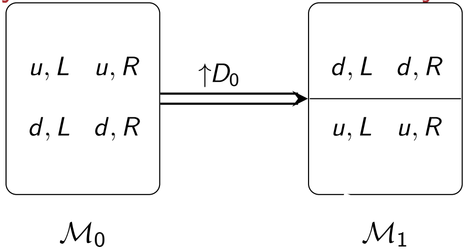

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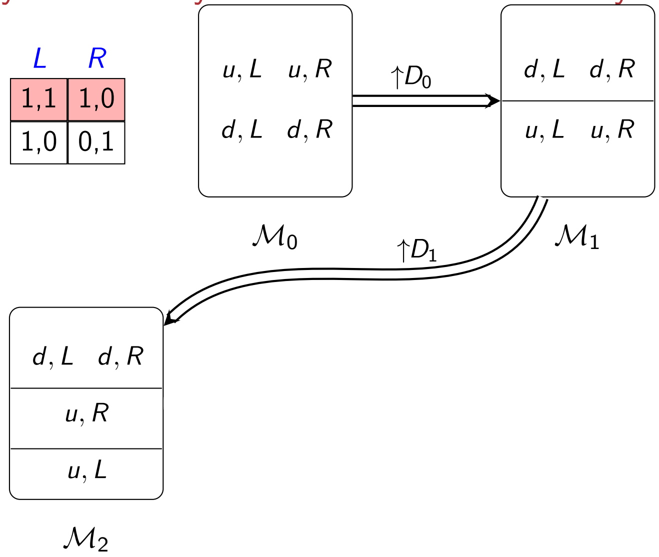

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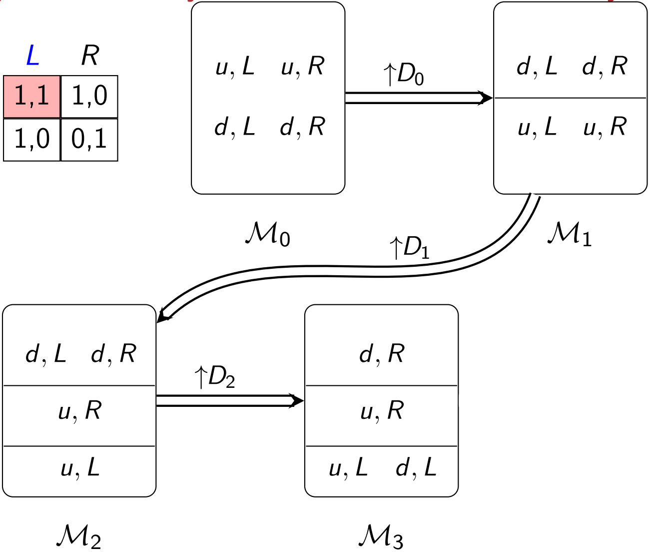

A Dynamic Analysis of Iterated Admissibility

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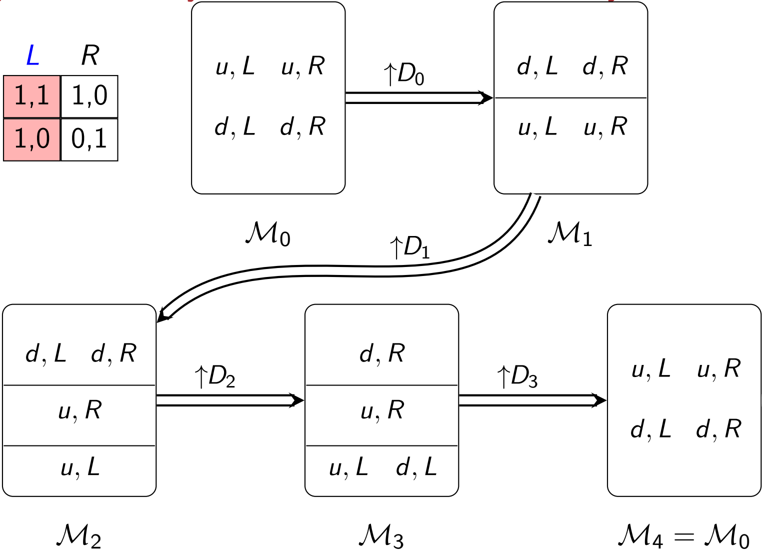

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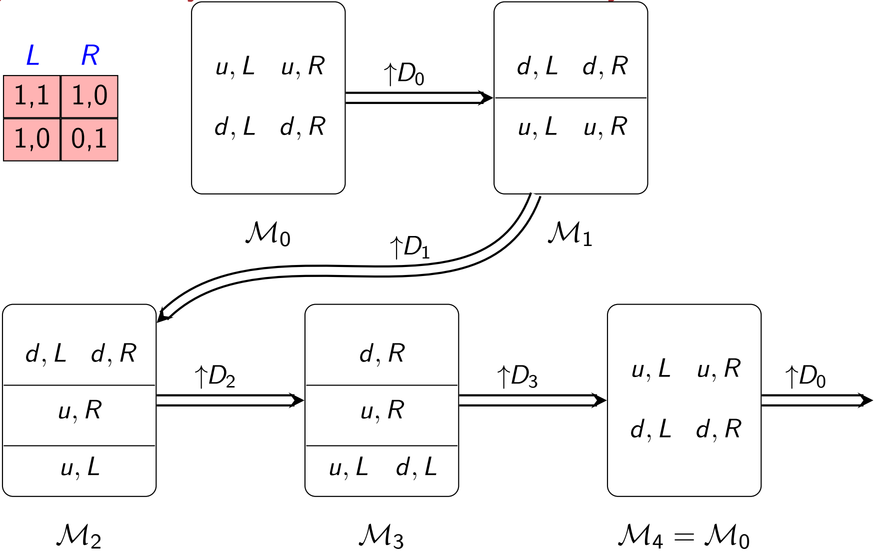
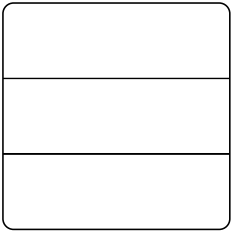


Suspending Judgement

Both φ_1 and φ_2 describe "good" options...



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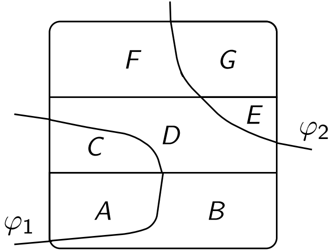

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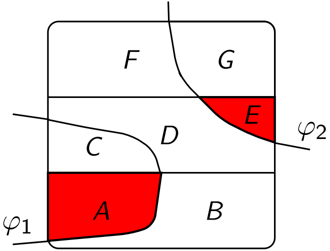

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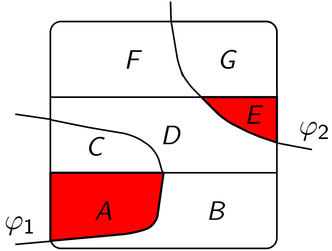

$$\uparrow\{\varphi_1, \varphi_2\} : A \cup E \prec B \prec C \cup D \prec F \cup G$$

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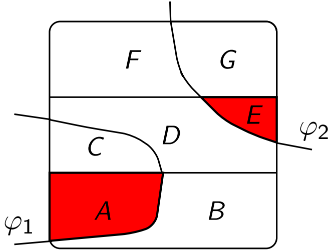

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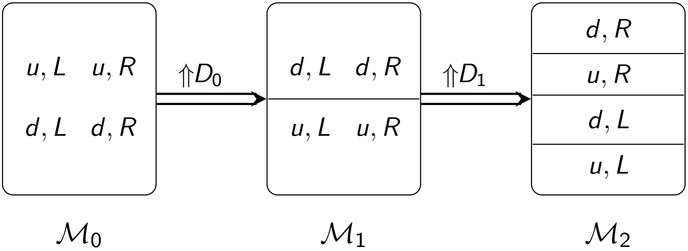
Remembering Reasons

	L	R
u	1,1	1,0
d	1,0	0,1



Game Plan

✓ Introduction, Motivation and Background

✓ The Dynamics of Rational Deliberation

✓ Reasoning to a Solution: Common Modes of Reasoning in Games

✓ Reasoning to a Model: Iterated Belief Change as Deliberation

Lecture 5: Reasoning in Specific Games

Pure Coordination

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	1,1

Hi-Low

		Bob L	R
Ann	U	3,3	0,0
	D	0,0	1,1

Focal Points

> *"There are these two broad empirical facts about Hi-Lo games, people almost always choose A [Hi] and people with common knowledge of each other's rationality think it is obviously rational to choose A [Hi]."*
>
> [Bacharach, *Beyond Individual Choice*, 2006, pg. 42]

See also chapter 2 of:

C.F. Camerer. *Behavioral Game Theory*. Princeton UP, 2003.

N. Bardsley, J. Mehta, C. Starmer and R. Sugden. *The Nature of Salience Revisited: Cognitive Hierarchy Theory* versus *Team Reasoning*. Economic Journal.

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(*level-n* theory/ *cognitive hierarchy* theory

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(*level-n* theory/ *cognitive hierarchy* theory

‘team reasoning’: assumes that each player chooses the decision rule which, if used by all players, would be optimal for each of them.

Do the two approaches make different predictions?

What do the experiments support?

pickers: choose between labels without any incentive to choose one rather than the other

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guessers: guess how pickers have behaved

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labels vs. *options*

$$\{water, beer, sherry, whisky, wine\}$$

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Task 1: pick an option

$\{\textbf{water}, \textit{beer}, \textit{sherry}, \textit{whisky}, \textit{wine}\}$

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Task 1: pick an option

Task 2: guess what your opponent picked

$\{$**water**, *beer*, *sherry*, *whisky*, *wine*$\}$

Task 1: pick an option
Task 2: guess what your opponent picked
Task 3: try to coordinate with your (unknown) partner

{**water**, *beer*, *sherry*, *whisky*, *wine*}

Task 1: pick an option
Task 2: guess what your opponent picked
Task 3: try to coordinate with your (unknown) partner

	pick	guess	coordinate
water	20	15	38
beer	13	26	11
sherry	4	1	0
whisky	6	6	5
wine	10	4	2

coordination game: the normal form plus the mechanism of labeling which allows players to distinguish between strategies.

$S_1 = \{s_{11}, \ldots, s_{1n}\}$
$S_2 = \{s_{21}, \ldots, s_{2n}\}$

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$U_1 = \cdots = U_n$ (pure coordination); otherwise a hi-low game.

$L = \{l_1, \ldots, l_n\}$ a set of distinct labels, common to both players.
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coordination index: For each l_j, let m_j be the number of individuals who choose it.

$$c = \sum_j \frac{m_j(m_j - 1)}{N(N-1)}$$

measure the probability that two distinct individuals, chosen at random without replacement from the set N of individuals, choose the same label.

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Why are *NCI*'s consistently higher than 1?

Players behavior is a probability on L: $\mathbf{p} = (p_1, \ldots, p_n)$

cognitive levels $k = 0, 1, 2, \ldots$

For each k, q_k is the relative frequency of level k players in the population.

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Level 2: With probability $q_0/(q_0 + q_1)$ the opponent reasons at level 0 and hence chooses according to $\mathbf{p}^0$. With probability $q_1/(q_0 + q_1)$ the opponent is a level 1 reasoner and choose l^* with probability 1. The level 2 reasoner chooses whichever label l^{**} maximizes this belief.

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$\mathbf{p}^1$ is not a **belief**, it is a probability distribution over the players possible beliefs about primary salience.

What is the relationship between $\mathbf{p}^0$ and $\mathbf{p}^1$?

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describable games

Hypothesis PC1: In any pure coordination game, the distribution of responses is at least as concentrated for guessers as it is for pickers. If the game is describable, the distribution is more concentrated for guessers.

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Hypothesis PC2: In any pure coordination game, the distribution of responses is the same for coordinators as for guessers.

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Interpret an option as a 'rule of selection'.

Primary and secondary salience can be used as rules of selection. Choose the label as if you were just picking" "Choose the label most likely to be picked by someone who is just picking".

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Hypothesis PC3: In any pure coordination game, if the guessing and coordination treatments generate different distributions of responses, the distribution from the coordination treatment is at least as concentrated as that from the guessing treatment.

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nondescript games: though normal players are aware that the labels are not the same, they do not have any readily-available way of describing those differences, even to themselves.

Schelling claims: "[I]f no better means of coordination can be discerned, the "solution" may be the strategy pair ... with payoffs of 9 apiece".

Choose label with payoff that is the odd one out

vs.

Choose label with payoff that is the highest payoff

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Team reasoning prefers the first strategy.

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Cognitive hierarchy: l_4 is chosen with probability $q_0/4$ and the others with probability $q_0/4 + (1 - q_0)/3$.

Hypothesis HL1: In any nondescript Hi-Lo game, the choice probability for each of the labels associated with the highest payoff is greater than that for every label associated with a lower payoff.

Hypothesis HL2: In any nondescript Hi-Lo game, the choice probability for each team-optimal label is greater than that for every other label.

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“ The implication is that our subjects were able to use subtle features of the experimental environment to solve the problem of coordinating on a common mode of reasoning. This behaviour reveals an ability to solve coordination problems at a conceptual level above that of the theories of cognitive hierarchy and team reasoning that we have been examining. Each of those theories captures certain aspects of focal-point reasoning, but some essential feature of the human ability to solve coordination problems seems to have escaped formalisation.”

"The basic intellectual premise, or working hypothesis, for rational players in this game seems to be the premise that some rule must be used if success is to exceed coincidence, and that the best rule to be found, whatever its rationalization, is consequently a rational rule." (Thomas Schelling)

Concluding Remarks: Reasoning in Games

"*The* fundamental insight of game theory [is] that a rational player must take into account that the players reason about each other in deciding how to play" (pg. 81)

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Exactly *how* the players incorporate the fact that they are interacting with other (actively reasoning) rational agents is the subject of much debate.

Concluding Remarks: Models of Strategic *Reasoning*

- Brian Skyrms' models of "dynamic deliberation"
- Ken Binmore's analysis using Turing machines to "calculate" the rational choice
- Robin Cubitt and Robert Sugden's "common modes of reasoning"
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Different frameworks, common thought: *the "rational solutions" of a game are the result of individual deliberation about the "rational" action to choose.*

Concluding Remarks: Higher-Order Beliefs

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It is true that a subjectivist Bayesian (in games) will have an opinion not only on his opponent's behavior, but also on his opponent's belief about his own behavior, his opponent's belief about his belief about his opponent's behavior, etc.

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- represent the *outcome* of a reasoning process: the *reasons* rational players can point to in order to justify their choices
- track the back-and-forth reasoning that players are engaged in as they deliberate about what to do

What are the players deliberating/reasoning *about*?

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Their preferences?

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Foundational issues: value of information, deliberation in decision theory, iterated belief change, the nature of practical deliberation

Thank you!

Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

$b^{A}_{C,S}$: A bet on proposition A that costs C to buy and pays S if won.

A bet is fair if, and only if, the agent is prepared to take each side of the bet (buy it, if offered, and sell it, if asked).

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Identification of credences with betting rates: $P(A) = C/S$

$$EU(\text{Buy } b^A_{C,S}) = P(A)\cdot(S-C) + P(\overline{A})(-C)$$

$$EU(\text{Buy } b^A_{C,S}) = P(A)\cdot(S-C) + P(\overline{A})(-C) =$$
$$C/S\cdot(S-C) - C/S\cdot C$$

$EU(\text{Buy } b^A_{C,S}) = P(A) \cdot (S - C) + P(\overline{A})(-C) =$
$C/S \cdot (S - C) - C/S \cdot C = 0$

$$EU(\text{Buy } b^A_{C,S}) = P(A)\cdot(S-C) + P(\overline{A})(-C) = C/S\cdot(S-C) - C/S\cdot C = 0$$

$$EU(\text{Sell } b^A_{C,S}) = P(A)\cdot(-S+C) + P(\overline{A})(C)$$

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Suppose that A and B are alternative actions available to the agent.

$EU(A)$ and $EU(B)$ are their expected utilities for the agent *disregarding any bets that he might place on the actions themselves*.

The "gain" G for an agent who accepts and wins a bet $b^A_{C,S}$ is the *net gain* $S - C$.

If he takes a bet on A with a net gain G, his expected utility of A will instead be $EU(A) + G$. The reason is obvious: If that bet is taken, then, if A is performed, the agent will receive G in addition to $EU(A)$.

"The agents readiness to accept a bet on an act does not depend on the betting odds but only on his gain. If the gain is high enough to put this act on the top of his preference order of acts, he will accept it, and if not, not. The stake of the agent is of no relevance whatsoever." (Spohn, 1977, p. 115)

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Take the bet "I will do action A" provided $EU(A) + G > EU(B)$ and if not, do not take the bet. *This has nothing to do with the ratio* C/S.

Suppose $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$ iff $G > 4$.

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The agent is certain that if he takes the bet on doing the action, then he will do that action.

Betting on an action is not the same thing as deciding to do an action.

▸ Skip

Argument II

Suppose that $P(A)$ is well-defined and $EU(A) < EU(B)$, but $EU(A) + G > EU(B)$

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If no bet on A is offered, then the agent does not think it is probable that he will perform A, so $P(A)$ is relatively low.

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Thus, the probability of an action depends on whether the bet is offered or not.

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forgetfulness

We must choose between the following 4 complex options:

1. take the bet on A & do A
2. take the bet on A & do B
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Claim 1: If an agent is certain that he won't perform an option, then this option is not *feasible*

Claim 2: If the agent assigns probabilities to options, then, on pain of incoherence, his probabilities for inadmissible (= irrational) options, as revealed by his betting dispositions, must be zero.

Consider two alternatives A and B with $EU(A) > EU(B)$.

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Suppose that the agent is offered a fair bet b on A, with a positive stake S and a price C. Since b is fair, $C/S = x$. Since $1 \geq x \geq 0$ and $S > 0$, it follows that $S \geq C \geq 0$.

Thus, $G = S - C \geq 0$.

Expected utilities of the complex actions:

- $EU(b \,\&\, A) = EU(A) + G$
- $EU(\neg b \,\&\, A) = EU(A)$
- $EU(b \,\&\, B) = EU(B) - C$
- $EU(\neg b \,\&\, B) = EU(B)$

At least one of $b \,\&\, A$ and $\neg b \,\&\, B$ is admissible.

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But then it follows that the agent should be willing to accept the bet on A even if $S = C$. Thus, the (fair) betting rate x for A must equal 1 ($P(A) = 1$), Which implies, on pain of incoherence, that $P(B) = 1 - P(A) = 0$. The inadmissible option has probability zero.

Do we have to conclude that probabilities for ones current options must lack any connection at all to ones potential betting behavior?

Rabinowicz: Suppose that the agent is offered an opportunity to make a *betting commitment* with respect to A at stake S and price C. The agent makes a commitment (to buy or sell) not knowing whether he will be required to sell or to buy the bet.

A betting commitment is *fair* if the agent is willing to accept the commitment even if he is *radically uncertain* about what will be required of him.