Models of Strategic Reasoning
Lecture 1

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August 6, 2012

Lecture 1: Introduction, Motivation and Background

Lecture 2: The Dynamics of Rational Deliberation

Lecture 3: Reasoning to a Solution: Common Modes of Reasoning in Games

Lecture 4: Reasoning to a Model: Iterated Belief Change as Deliberation

Lecture 5: Reasoning in Specific Games: Experimental Results

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`ai.stanford.edu/~epacuit/esslli2012/stratreas.html`

Plan for Today

- Introductory Remarks
- Equilibrium Selection and Harsanyi's tracing procedure
- Higher-order beliefs in games
- General comments on common knowledge and beliefs

The "Axiom" of Game Theory

Common Knowledge of Rationality

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“it is completely *transparent* to the players that...”

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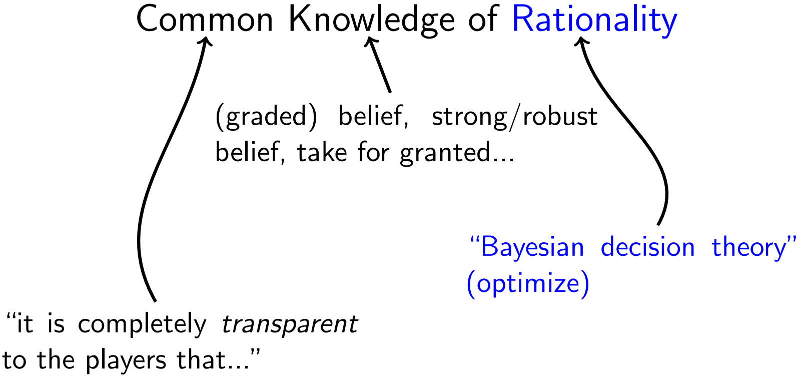

Common Knowledge of Rationality

(graded) belief, strong/robust belief, take for granted...

“Bayesian decision theory” (optimize)

“it is completely *transparent* to the players that...”

Dynamics

"The economist's predilection for equilibria frequently arises from the belief that some underlying dynamic process (often suppressed in formal models) moves a system to a point from which it moves no further." (pg. 1008)

B. D. Bernheim. *Rationalizable Strategic Behavior*. Econometrica, 52, 4, pgs. 1007 - 1028.

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"It is not just a question of what common knowledge obtains at the moment of truth, but also how common knowledge is preserved, created, or destroyed in the deliberational process which leads up to the moment of truth." (pg. 160)

B. Skyrms. *The Dynamics of Rational Deliberation*. Harvard University Press, 1990.

Substantive vs. Procedural Rationality

"Behavior is **substantively rational** when it is appropriate to the achievement of given goals within the limits imposed by given conditions and constraints. [...] Given these goals, the rational behavior is determined entirely by the characteristics of the environment in which it takes place. "

H. Simon. *From Substantive to Procedural Rationality*. in Method and Appraisal in Economics, 1976.

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"Behavior is **substantively rational** when it is appropriate to the achievement of given goals within the limits imposed by given conditions and constraints. [...] Given these goals, the rational behavior is determined entirely by the characteristics of the environment in which it takes place. "

"Behavior is **procedurally rational** when it is the outcome of appropriate deliberation. Its procedural rationality depends on the process that generated it. When psychologists use the term rational, it is usually procedural rationality they have in mind. "

H. Simon. *From Substantive to Procedural Rationality*. in Method and Appraisal in Economics, 1976.

Substantive vs. Procedural Rationality, II

"The human mind is programmable: it can acquire an enormous variety of different skills, behavior patterns, problem-solving repertoires, and perceptual habits. [...]

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H. Simon. *From Substantive to Procedural Rationality*. in Method and Appraisal in Economics, 1976.

Rationality: Two Themes

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Rationality is a matter of **reasons**:

- The rationality of a belief P depends on the *reasons for holding* P
- The rationality of act α depends on the *reason for doing* α

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Rationality is a matter of **reliability**:

- A rational belief is one that is arrived at a through a process that reliably produces beliefs that are true.
- A act is rational if it is arrived at through a process that reliably achieves specified goals.

Rationality: Two Themes

"Neither theme alone exhausts our notion of rationality. Reasons without reliability seem emtpy, reliability without reasons seems blind. In tandem these make a powerful unit, but how exactly are they related and why?" (Nozick, pg. 64)

R. Nozick. *The Nature of Rationality*. Princeton University Press, 1993.

From Game Models to Models of Games

A **game** is a description of strategic interaction that includes

- actions the players *can* take
- description of the players' interests (i.e., preferences),
- description of the "structure" of the decision problem

From Game Models to Models of Games

"We adhere to the classical point of view that the game under consideration fully describes the real situation — that any (pre) commitment possibilities, any repetitive aspect, any probabilities of error, or any possibility of jointly observing some random event, have already been modeled in the game tree." (pg. 1005)

E. Kohlberg and J.-F. Mertens. *On the strategic stability of equilibria*. Econometrica, 54, pgs. 1003 - 1038, 1986.

From Game Models to Models of Games

“ Formally, a game is defined by its strategy sets and payoff functions. But in real life, many other parameters are relevant; there is a lot more going on. Situations that substantively are vastly different may nevertheless correspond to precisely the same strategic game. For example, in a parliamentary democracy with three parties, the winning coalitions are the same whether the parties each hold a third of the seats in parliament, or, say, 49 percent, 39 percent, and 12 percent, respectively. But the political situations are quite different. The difference lies in *the attitudes of the players, in their expectations about each other, in custom, and in history*, though the rules of the game do not distinguish between the two situations. (pg. 72, our emphasis)

R. Aumann, Robert and J. Dreze. *Rational expectations in games*. American Economic Review, 98(1): 72 86, 2008..

Epistemic Program in Game Theory

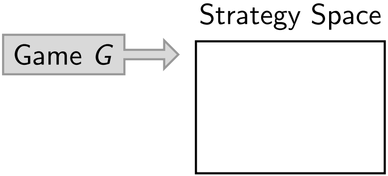
A. Brandenburger. *The Power of Paradox*. International Journal of Game Theory, 35, pgs. 465 - 492, 2007.

EP and O. Roy. *Epistemic Game Theory*. Stanford Encyclopedia of Philosophy, forthcoming, 2012.

Epistemic Program in Game Theory

Game G

Epistemic Program in Game Theory



Strategy Space

Game G

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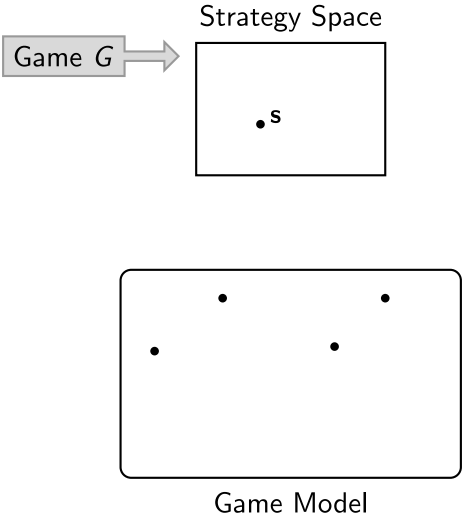

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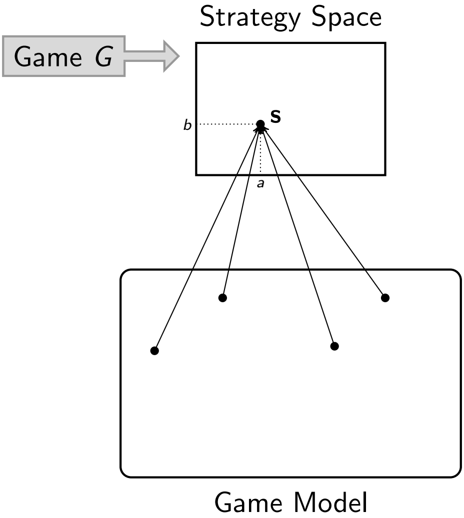

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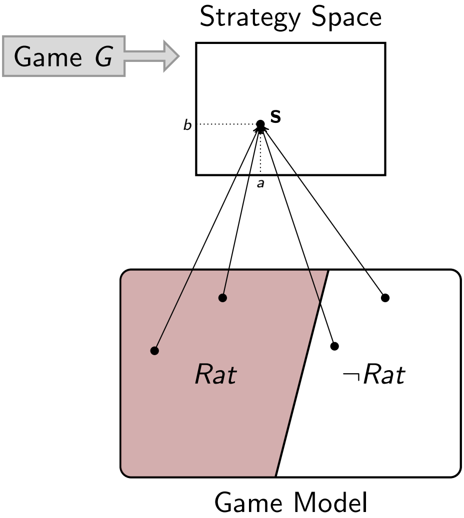

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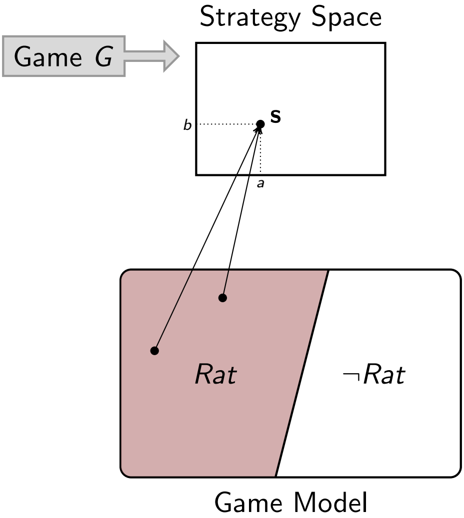
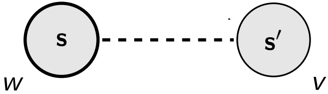


Models of Hard and Soft Information



Epistemic Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$

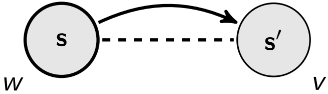
- $w \sim_i v$ means i cannot rule out v according to her information.

Language: $\varphi := p \mid \neg\varphi \mid \varphi\wedge\psi \mid K_i\varphi$

Truth:

- $\mathcal{M}, w \models p$ iff $w \in V(p)$ (p an atomic proposition)
- Boolean connectives as usual
- $\mathcal{M}, w \models K_i\varphi$ iff for all $v \in W$, if $w \sim_i v$ then $\mathcal{M}, v \models \varphi$

Models of Hard and Soft Information



Epistemic-Plausibility Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\preceq_i\}_{i\in\mathcal{A}}, V\rangle$

- $w \preceq_i v$ means v is at least as plausibility as w for agent i.

Language: $\varphi := p \mid \neg\varphi \mid \varphi \wedge \psi \mid K_i\varphi \mid B^{\varphi}\psi \mid [\preceq_i]\varphi$

Truth:

- $[\![\varphi]\!]_{\mathcal{M}} = \{w \mid \mathcal{M}, w \models \varphi\}$
- $\mathcal{M}, w \models B_i^{\varphi}\psi$ iff for all $v \in Min_{\preceq_i}([\![\varphi]\!]_{\mathcal{M}} \cap [w]_i)$, $\mathcal{M}, v \models \psi$
- $\mathcal{M}, w \models [\preceq_i]\varphi$ iff for all $v \in W$, if $v \preceq_i w$ then $\mathcal{M}, v \models \varphi$

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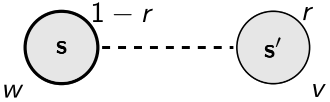

Epistemic-Plausibility Model: $\mathcal{M} = \langle W, \{\sim_i\}_{i\in\mathcal{A}}, \{\pi_i\}_{i\in\mathcal{A}}, V\rangle$

- $\pi_i : W \to [0,1]$ is a probability measure

Language: $\varphi := p \mid \neg\varphi \mid \varphi\wedge\psi \mid K_i\varphi \mid B^p\psi$

Truth:

- $[\![\varphi]\!]_{\mathcal{M}} = \{w \mid \mathcal{M}, w \models \varphi\}$
- $\mathcal{M}, w \models B^p\varphi$ iff $\pi_i([\![\varphi]\!]_{\mathcal{M}} \mid [w]_i) = \frac{\pi_i([\![\varphi]\!]_{\mathcal{M}}\cap[w]_i)}{\pi_i([w]_i)} \geq p$, $\mathcal{M}, v \models \psi$
- $\mathcal{M}, w \models K_i\varphi$ iff for all $v \in W$, if $w \sim_i v$ then $\mathcal{M}, v \models \varphi$

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From Substantive to Procedural Rationality

"Discussions of substantive rationality take place in an essentially *static* framework. Thus, equilibrium is discussed without explicit reference to any dynamic process by means of which the equilibrium is achieved. Similarly, prior beliefs are taken as given, without reference to how a rational agent acquires these beliefs. Indeed, all questions of the procedure by means of which rational behavior is achieved are swept aside by a methodology that treats this procedure as completed and reifies the supposed limiting entities by categorizing them axiomatically." (pg. 180)

K. Binmore. *Modeling Rational Players: Part I*. Economics and Philosophy, 3, pgs. 179 - 214, 1987.

From Substantive to Procedural Rationality

What does it mean to choose "rationally"?

"A glance at any dictionary will confirm that economists, firmly entrenched in the static viewpoint described above, have hijacked this word and used it to mean something for which the word *consistent* would be more appropriate. " (pg. 181)

K. Binmore. *Modeling Rational Players: Part I*. Economics and Philosophy, 3, pgs. 179 - 214, 1987.

Models of Strategic *Reasoning*

- Brian Skyrms' models of "dynamic deliberation"
- Ken Binmore's analysis using Turing machines to "calculate" the rational choice
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Different frameworks, common thought: *the "rational solutions" of a game are the result of individual deliberation about the "rational" action to choose.*

Reasoning in Games: A Heuristic Treatement

Background: Setting the Stage

(Subjective) Probability ("betting interpretation"), Von Neumann-Morgenstern Utilities, Expected Utility, Extensive/Strategic Games, Nash Equilibrium

Suppose there are two players Ann and Bob dividing a cake. Suppose that Ann cuts the cake and then Bob chooses the first piece. (Suppose they *only* care about the size of the piece). Ann cannot cut the cake exactly evenly, so one piece is always larger than the other.

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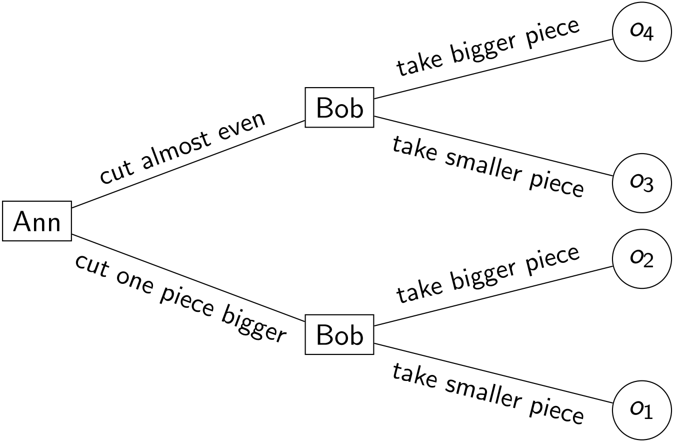

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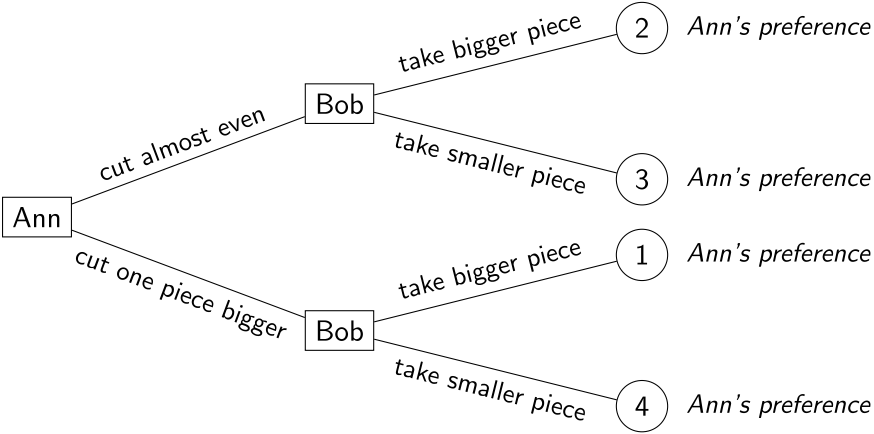

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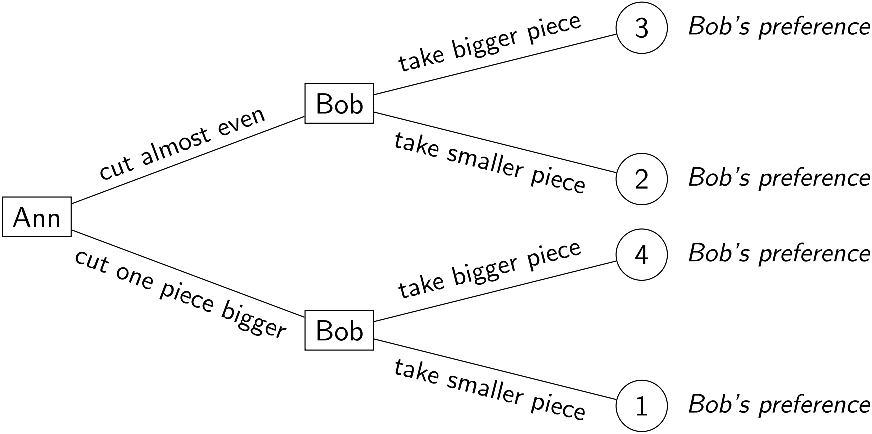
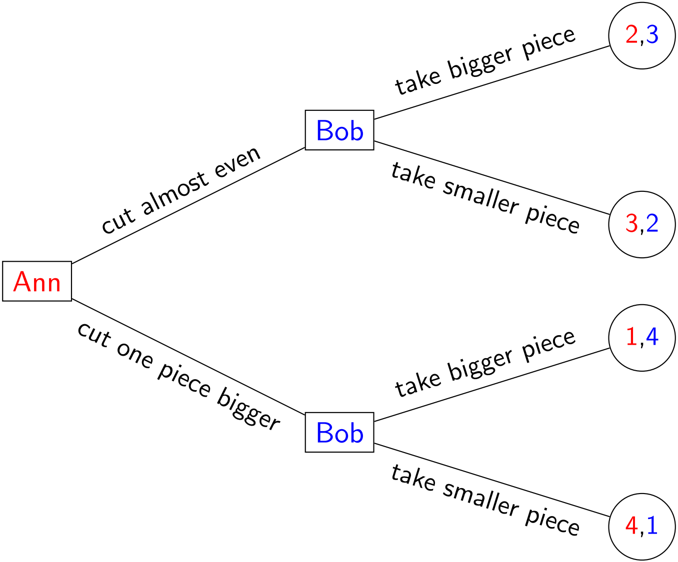
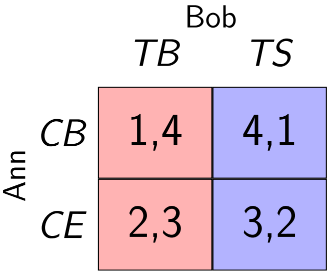



Ann
cut almost even
cut one piece bigger
Bob
Bob
take bigger piece
take smaller piece
take bigger piece
take smaller piece
2,3
3,2
1,4
4,1

		Bob	
		TB	*TS*
Ann	*CB*	1,4	4,1
	CE	2,3	3,2

What should Ann *do*?



		Bob	
		TB	*TS*
Ann	*CB*	1,4	4,1
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What should Ann *do*? *Bob best choice in Ann's worst choice*

		Bob		
		TB	*TS*	
Ann	*CB*	1,4	4,1	1
	CE	2,3	3,2	2

What should Ann *do*? *maximize over each row and choose the maximum value*

		Bob	
		TB	*TS*
Ann	*CB*	1,4	4,1
	CE	2,3	3,2
		3	1

What should Bob *do*? *minimize over each column and choose the maximum value*

Theorem (Von Neumann). For every two-player zero- sum game with finite strategy sets S_1 and S_2, there is a number v, called the **value** of the game such that:

1. $v = \max_{p\in\Delta(S_1)} \min_{q\in\Delta(S_2)} U_1(p,q) =$ $\min_{q\in\Delta(S_2)} \max_{p\in\Delta(S_1)} U_1(p,q)$
2. The set of mixed Nash equilibria is nonempty. A mixed strategy profile (p,q) is a Nash equilibrium if and only if

$$p \in \mathrm{argmax}_{p\in\Delta(S_1)} \min_{q\in\Delta(S_2)} U_1(p,q)$$

$$q \in \mathrm{argmax}_{q\in\Delta(S_2)} \min_{p\in\Delta(S_1)} U_1(p,q)$$

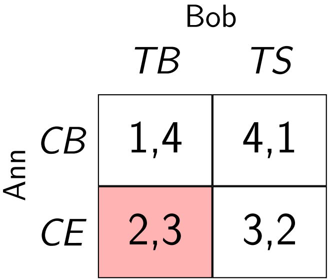
3. For all mixed Nash equilibria (p,q), $U_1(p,q) = v$

Why play such an equilibrium?

"Let us now imagine that there exists a complete theory of the zero-sum two-person game which tells a player what to do, and which is absolutely convincing. If the players knew such a theory then each player would have to assume that his strategy has been "found out" by his opponent. The opponent knows the theory, and he knows that the player would be unwise not to follow it... a satisfactory theory can exist only if we are able to harmonize the two extremes...strategies of player 1 'found out' or of player 2 'found out.' " (pg. 148)

J. von Neumann and O. Morgenstern. *Theory of Games and Economic Behavior*. Princeton University Press, 1944.

> "Von Neumann and Morgenstern are assuming that the *payoff matrix* is common knowledge to the players, but presumably the players' subjective probabilities might be private. Then each player might quite reasonably act to maximize subjective expected utility, believing that he will *not* be found out, with the result *not* being a Nash equilibrium."
>
> (Skyrms, pg. 14)



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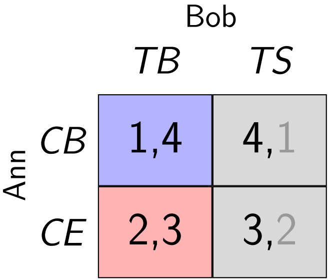
- Suppose that Ann believes Bob will play *TB* with probability $1/4$, *for whatever reason*. Then,

$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$



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$$1 \times 0.25 + 4 \times 0.75 = 3.25 \geq 2 \times 0.25 + 3 \times 0.75 = 2.75$$

- But, *TB* is maximizes expected utility no matter what belief Bob may have:

$$p + 3 = 4 \times p + 3 \times (1 - p) \geq 1 \times p + 2 \times (1 - p) = 2 - p$$

“The rules of a game and its numerical data are seldom sufficient for logical deduction alone to single out a unique choice of strategy for each player. ‘**To do so one requires either richer information (such as institutional detail or perhaps historical precedent for a certain type of behavior) or bolder assumptions about how players choose strategies.** Putting further restrictions on strategic choice is a complex and treacherous task. But one’s intuition frequently points to patterns of behavior that cannot be isolated on the grounds of consistency alone.” (pg. 1035)

D. G. Pearce. *Rationalizable Strategic Behavior*. Econometrica, 52, 4, pgs. 1029 - 1050, 1984.

Finding the rational choice...

		Bob	
		H	*T*
Ann	*H*	1,-1	-1,1
	T	-1,1	1,-1

What is a rational choice for Ann (Bob)?

Finding the rational choice...

		Bob	
		H	*T*
Ann	*H*	1,-1	-1,1
	T	-1,1	1,-1

What is a rational choice for Ann (Bob)? *Flip a coin!*

Finding the rational choice...

		Bob	
		$C1$	$C2$
Ann	$P1$	1,-1	-1,1
	$P2$	-1,1	1,-1

What is a rational choice for Ann (Bob)?

Finding the rational choice...

		Bob	
		$C1$	$C2$
Ann	$P1$	1,-1	-1,1
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		Bob	
		$C1$	$C2$
Ann	$P1$	1,-1	1,-1
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What is a rational choice for Ann (Bob)? *Play a different game!*

Prisoner's Dilemma

Two people commit a crime.

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Two people commit a crime. The are arrested by the police, who are quite sure they are guilty but cannot prove it without at least one of them confessing. The police offer the following deal. Each one of them can confess and get credit for it. If only one confesses, he becomes a state witness and not only is he not punished, he gets a reward. If both confess, they will be punished but will get reduced sentences for helping the police. If neither confesses, the police honestly admit that there is no way to convict them, and they are set free.

Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Prisoner's Dilemma

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Possible outcomes:

Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess (C, C),

Prisoner's Dilemma

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Possible outcomes: We both confess (C, C), I confess but my partner doesn't (C, D),

Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess (C, C), I confess but my partner doesn't (C, D), My partner confesses but I don't (D, C),

Prisoner's Dilemma

Two options: Confess (C), Don't Confess (D)

Possible outcomes: We both confess (C, C), I confess but my partner doesn't (C, D), My partner confesses but I don't (D, C), neither of us confess (D, D).

Prisoner's Dilemma

		Bob	
		D	C
Ann	D		
	C		

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3	1
	C	4	2

Ann's preferences

Prisoner's Dilemma

Bob

		D	*C*
Ann	*D*	3	4
	C	1	2

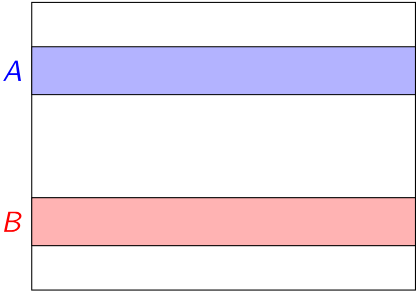
Bob's preferences

Prisoner's Dilemma

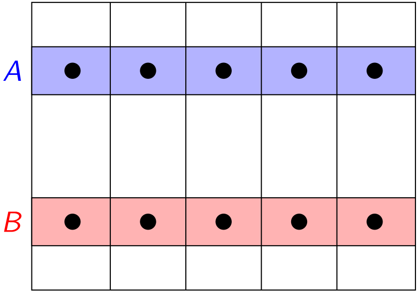
		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do?

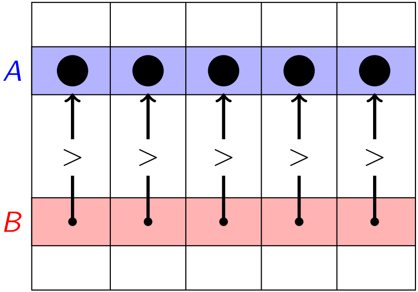
Dominance Reasoning



Dominance Reasoning



Dominance Reasoning



Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do?

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning*

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Dominance reasoning* is not **Pareto**!

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3	2.5
	C	2.5	2

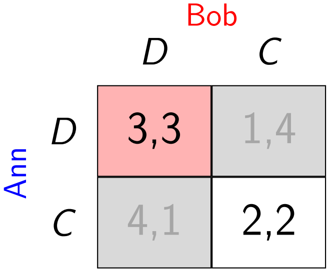
What should Ann (Bob) do? *Think as a group*!

Prisoner's Dilemma

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma



		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Play against your mirror image*!

Prisoner's Dilemma

		Bob	
		D	C
Ann	D	ϵ,ϵ	1,4
	C	4,1	2,2

What should Ann (Bob) do? *Change the game* (eg., Symbolic Utilities)

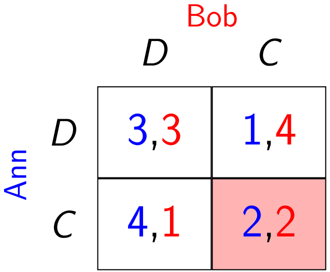
		Bob	
		D	C
Ann	D	4,4	1,3
	C	3,1	2,2

What should/will Ann (Bob) do?

		Bob	
		D	*C*
Ann	*D*	4,4	1,3
	C	3,1	2,2

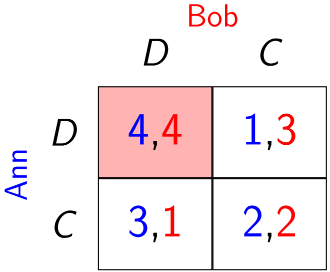
Assurance Game

What should/will Ann (Bob) do?



		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma



		Bob	
		D	C
Ann	D	4,4	1,3
	C	3,1	2,2

Assurance Game

What should/will Ann (Bob) do?

Nozick: Symbolic Utility

“Yet the symbolic value of an act is not determined solely by *that* act.

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"Yet the symbolic value of an act is not determined solely by *that* act. The act's meaning can depend upon what other acts are available with what payoffs and what acts also are available to the other party or parties.

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R. Nozick. *The Nature of Rationality*. Princeton University Press, 1993.

Bob

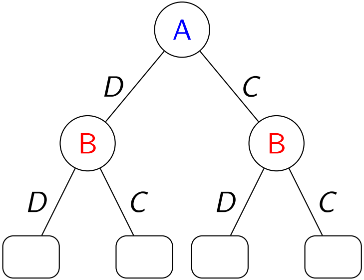
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma

What should/will Ann (Bob) do?

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

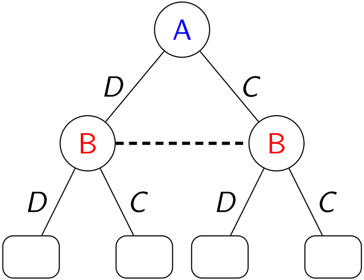
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma



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		Bob	
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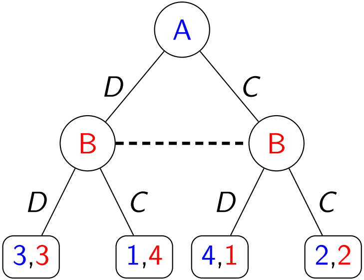
Ann	*D*	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma



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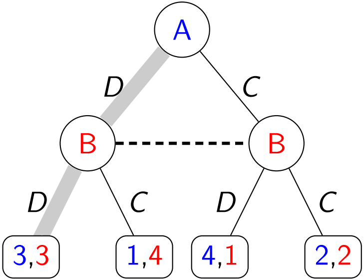
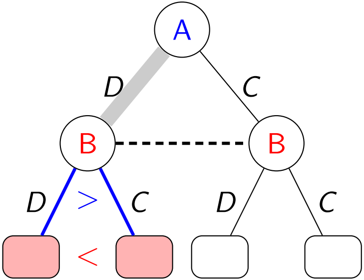


What should/will Ann (Bob) do?

		Bob	
		D	C
Ann	D	3,3	1,4
	C	4,1	2,2

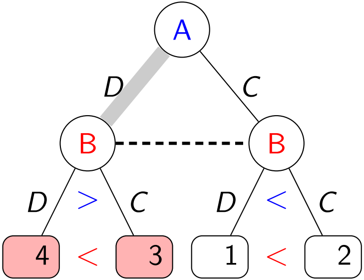
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma



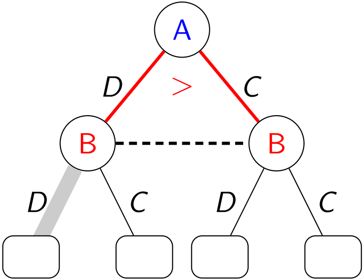
What should/will Ann (Bob) do?

Bob

Ann

	D	*C*
D	3,3	1,4
C	4,1	2,2

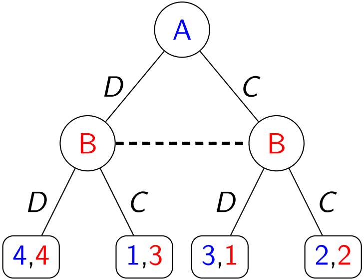
Prisoner's Dilemma



What should/will Ann (Bob) do?

		Bob	
		D	*C*
Ann	*D*	3,3	1,4
	C	4,1	2,2

Prisoner's Dilemma



What should/will Ann (Bob) do?

"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation.

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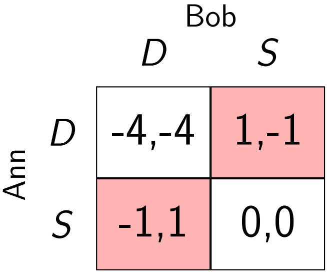
"Game theorists think it just plain wrong to claim that the Prisoners' Dilemma embodies the essence of the problem of human cooperation. On the contrary, it represents a situation in which the dice are as loaded against the emergence of cooperation as they could possibly be. If the great game of life played by the human species were the Prisoner's Dilemma, we wouldn't have evolved as social animals! .... No paradox of rationality exists. Rational players don't cooperate in the Prisoners' Dilemma, because the conditions necessary for rational cooperation are absent in this game." (pg. 63)

K. Binmore. *Natural Justice*. Oxford University Press, 2005.

Equilibrium are not interchangeable

		Bob	
		D	*S*
Ann	*D*	-4,-4	1,-1
	S	-1,1	0,0

Equilibrium are not interchangeable

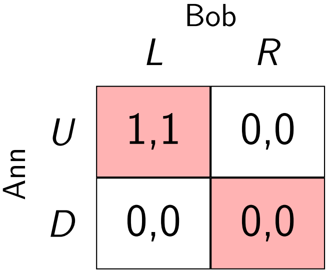


		Bob	
		D	*S*
Ann	*D*	-4,-4	1,-1
	S	-1,1	0,0

Perfect equilibrium

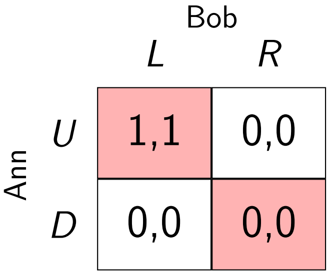
		Bob	
		L	*R*
Ann	*U*	1,1	0,0
	D	0,0	0,0

Perfect equilibrium



		Bob	
		L	*R*
Ann	*U*	1,1	0,0
	D	0,0	0,0

Perfect equilibrium



		Bob	
		L	*R*
Ann	*U*	1,1	0,0
	D	0,0	0,0

Isn't (U, L) more "reasonable" than (D, R)?

Perfect equilibrium

		Bob	
		L	*R*
Ann	*U*	1,1	0,0
	D	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

Perfect equilibrium

		Bob	
		L	R
Ann	U	1,1	0,0
	D	0,0	0,0

Completely mixed strategy: a mixed strategy in which every strategy gets some positive probability

ϵ-**perfect equilibrium**: a completely mixed strategy profile in which any pure strategy that is not a best reply receives probability less than ϵ

Prefect equilibrium: the mixed strategy profile that is the limit as ϵ goes to 0 of ϵ-prefect equilibria.

Proper equilibrium

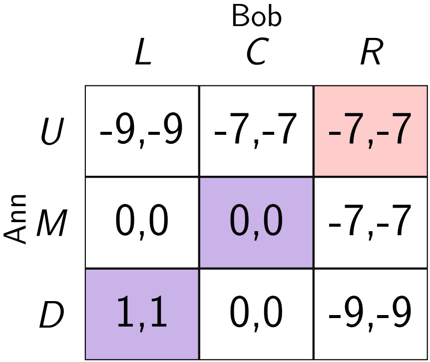
Bob

		L	*C*	*R*
	U	-9,-9	-7,-7	-7,-7
Ann	*M*	0,0	0,0	-7,-7
	D	1,1	0,0	-9,-9

Proper equilibrium

			Bob	
		L	*C*	*R*
	U	-9,-9	-7,-7	-7,-7
Ann	*M*	0,0	0,0	-7,-7
	D	1,1	0,0	-9,-9

Proper equilibrium



			Bob	
		L	*C*	*R*
	U	-9,-9	-7,-7	-7,-7
Ann	*M*	0,0	0,0	-7,-7
	D	1,1	0,0	-9,-9

ϵ-**proper equilibrium**: a completely mixed strategy profile such that if strategy s is a better response than s', then $\frac{p(s)}{p(s')} < \epsilon$

Proper equilibrium: the mixed strategy profile that is the limit as ϵ goes to 0 of ϵ-proper equilibria.

Proper equilibrium

			Bob	
		L	C	R
	U	-9,-9	-7,-7	-7,-7
Ann	M	0,0	0,0	-7,-7
	D	1,1	0,0	-9,-9

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Trembling Hands

“There cannot be any mistakes if the players are absolutely rational. Nevertheless, a satisfactory interpretation of equilibrium points in extensive games seems to require that the possibility of mistakes is not completely excluded. This can be achieved by a point of view which looks at complete rationality as the limiting case of incomplete rationality.” (pg. 35)

R. Selten. *Reexamination of the Perfectness Concept of Equilibrium in Extensive Games*. International Journal of Game Theory, 4, pgs. 25 - 55, 1975.

Tracing Procedure

J.C. Harsanyi. *The Tracing Procedure: A Bayesian Approach to Defining a Solution for n-Person Noncooperative Games*. International Journal of Game Theory, 4, pgs. 61 - 94, 1975.

J. C. Harsanyi and R. Selten. *A general theory of equilibrium selection in games*. The MIT Press, 1988.

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P. Jean-Jacques Herings. *Two simple proofs of the feasibility of the linear tracing procedure*. Economic Theory, 15, pgs. 485 - 490, 2000.

S. H. Schanuel, L.K. Simon, and W. R. Zame. *The algebraic geometry of games and the tracing procedure*. in *Game equilibrium models II: methods, morals and markets*, pgs. 9 - 43, Springer, 1991.

The Tracing Procedure

	L	R
U	4,1	0,0
D	0,0	1,4

The Tracing Procedure

	L	R
U	4,1	0,0
D	0,0	1,4

- Suppose there is a common prior that Ann will choose U with probability 0.5 and Bob will choose L with probability 0.5

The Tracing Procedure

	L	R
U	4,1	0,0
D	0,0	1,4

	L	R
U	2,0.5	2,2
D	0.5,0.5	0.5,2

- Suppose there is a common prior that Ann will choose U with probability 0.5 and Bob will choose L with probability 0.5
- Consider the modified game where the utilities are the *expected utilities* of the first game (G^1)

The Tracing Procedure

	L	R
U	4,1	0,0
D	0,0	1,4

	L	R
U	2,0.5	2,2
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- Suppose there is a common prior that Ann will choose U with probability 0.5 and Bob will choose L with probability 0.5
- Consider the modified game where the utilities are the *expected utilities* of the first game (G^1)
- This game as a unique Nash equilibrium

The Tracing Procedure

	L	R
U	4,1	0,0
D	0,0	1,4

	L	R
U	2,0.5	2,2
D	0.5,0.5	0.5,2

- For each $t \in [0, 1]$, the game G^t is defined so that the payoffs of $u_i^t(x, y) = x \cdot u_i^0(x, y) + (1 - x) \cdot u_i^1(x, y)$

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U	4,1	0,0
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- A graph of the equilibrium points as t varies from 0 to 1 will show a connected path from equilibria in G^0 to equilibria in G^1

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	L	R
U	2,0.5	2,2
D	0.5,0.5	0.5,2

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- A graph of the equilibrium points as t varies from 0 to 1 will show a connected path from equilibria in G^0 to equilibria in G^1
- This process almost always leads to a unique equilibrium in G^1 (modifying the payoffs with a a logarithmic term guarantees uniqueness)

Harsanyi and Selten also propose a method to *calculate* a prior given the game.

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They can identify game-theoretic equilibria instantaneously.

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They can identify game-theoretic equilibria instantaneously.

At $t = 0$ each contemplates jumping to the conclusion that the act the with maximum expected utility according to the common prior is the correct one.

At later times, the hypothesis that the other players will make their best response gets stronger and stronger, until at $t = 1$ only an equilibrium point of the original game remains.

Two general issues

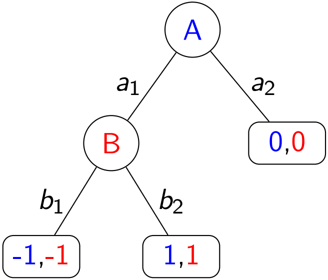
- The relation between sequential and normal form games/decisions
- What are the players deliberating/reasoning *about*?

▸ Conclusion

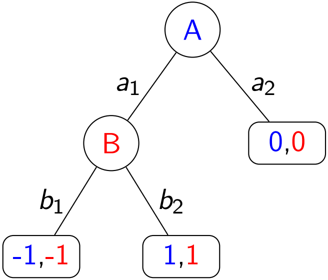
Normal form vs. Extensive Form

Explicitly modeling deliberation transforms a *single* choice into a situation of *sequential* choice.

Normal form vs. Extensive form

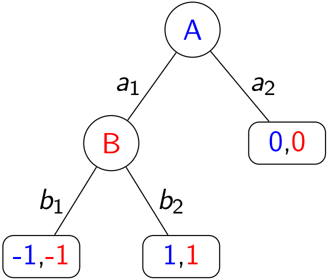


Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
a_1	-1,-1	1,1
a_2	0,0	0,0

Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
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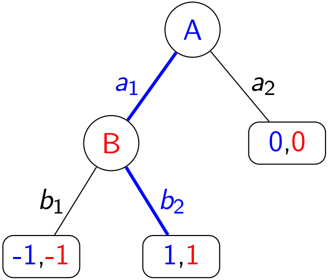
Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
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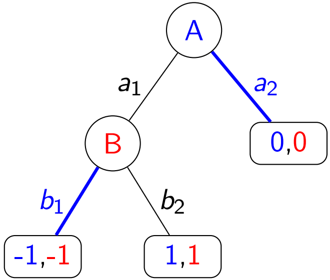
Normal form vs. Extensive form



	b_1 if a_1	b_2 if a_1
a_1	-1,-1	1,1
a_2	0,0	0,0

(Cf. the various notions of *sequential equilibrium*)

T. Seidenfeld. *When normal and extensive form decisions differ*. in *Logic, Methodology and Philosophy of Science IX*, Elsevier, 1994.

What are the players deliberating/reasoning *about*?

What are the players deliberating/reasoning *about*?

Their preferences?

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Their preferences? The model?

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Their preferences? The model? The other players?

What are the players deliberating/reasoning *about*?

Their preferences? The model? The other players? What to *do*?

▸ Conclusion

The Cost of Thinking

“A person required to risk money on a remote digit of π would have to compute that digit in order to comply fully with the theory, though this would really be wasteful if the cost of computation were more than the prize involved.

The Cost of Thinking

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Deliberation in Decision Theory

"deliberation crowds out prediction"

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▸ Conclusion

Meno’s Paradox

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Levi's Argument

1. If you have access to self-knowledge and logical omniscience to apply the principles of rational choice to determine which options are admissible, then the principles of rational choice are vacuous for the purposes of deciding what to do.
2. If you do not have access to self-knowledge and logical omniscience in this sense, then the principles of rational choice are inapplicable for the purposes of deciding what do.

Therefore, the principles of rational choice are either unnecessary or impossible.

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If all the previous conditions are satisfied, then no inadmissible option is feasible from the deliberating agent's point of view when deciding what to do: $C(A) = A$.

"Though this result is not contradictory, it implies the vacuousness of principles of rational choice for the purpose of deciding what to do...If they are useless for this purpose, then by the argument of the previous section, they are useless for passing judgement on the rationality of choice as well." (L, pg. 10)

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(Earlier argument: "If X is merely giving advice, it is pointless to advise Sam to do something X is sure Sam will not do...The point I mean to belabor is that passing judgement on the rationality of Sam's choices has little merit unless it gives advice to how one should choose in predicaments similar to Sam's in relevant aspects")

Weak Thesis: In a situation of choice, the DM does not assign extreme probabilities to options among which his choice is being made.

Strong Thesis: In a situation of choice, the DM does not assign any probabilities to options among which his choice is being made.

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"...the probability assignment to A may still be available to the subject in his purely doxastic capacity but not in his capacity of an agent or practical deliberator. The agent *qua* agent must abstain from assessing the probability of his options." (Rabinowicz, pg. 3)

“(...) probabilities of acts play no role in decision making. (...) The decision maker chooses the act he likes most be its probability as it may. But if this is so, there is no sense in imputing probabilities for acts to the decision maker.” (Spohn (1977), pg. 115)

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- Deliberation as a *feedback* process: change in inclinations causes a change in probabilities assigned to various options, which in turn may change my inclinations towards particular options....

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- Drop smugness: “the agent need not assume he will choose rationally...the agent should be in a state of suspense as to which of the feasible options will be chosen” (Levi)
- Implications for game theory (*common knowledge of rationality* implies, in particular, that agents satisfy *Smugness*).

▸ Conclusion

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$b^A_{C,S}$: A bet on proposition A that costs C to buy and pays S if won.

A bet is fair if, and only if, the agent is prepared to take each side of the bet (buy it, if offered, and sell it, if asked).

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Identification of credences with betting rates: $P(A) = C/S$

$$EU(\mathrm{Buy}\ b^{A}_{C,S}) = P(A)\cdot(S-C) + P(\overline{A})(-C)$$

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Suppose that A and B are alternative actions available to the agent.

$EU(A)$ and $EU(B)$ are their expected utilities for the agent *disregarding any bets that he might place on the actions themselves*.

The "gain" G for an agent who accepts and wins a bet $b^{A}_{C,S}$ is the *net gain* $S - C$.

If he takes a bet on A with a net gain G, his expected utility of A will instead be $EU(A) + G$. The reason is obvious: If that bet is taken, then, if A is performed, the agent will receive G in addition to $EU(A)$.

"The agents readiness to accept a bet on an act does not depend on the betting odds but only on his gain. If the gain is high enough to put this act on the top of his preference order of acts, he will accept it, and if not, not. The stake of the agent is of no relevance whatsoever." (Spohn, 1977, p. 115)

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Take the bet "I will do action A" provided $EU(A) + G > EU(B)$ and if not, do not take the bet. *This has nothing to do with the ratio C/S.*

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The agent is certain that if he takes the bet on doing the action, then he will do that action.

Betting on an action is not the same thing as deciding to do an aciton.

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If no bet on A is offered, then the agent does not think it is probable that he will perform A, so $P(A)$ is relatively low.

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Thus, the probability of an action depends on whether the bet is offered or not.

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forgetfulness

We must choose between the following 4 complex options:

1. take the bet on A & do A
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Claim 1: If an agent is certain that he won't perform an option, then this option is not *feasible*

Claim 2: If the agent assigns probabilities to options, then, on pain of incoherence, his probabilities for inadmissible (= irrational) options, as revealed by his betting dispositions, must be zero.

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Suppose that the agent is offered a fair bet b on A, with a positive stake S and a price C. Since b is fair, $C/S = x$. Since $1 \geq x \geq 0$ and $S > 0$, it follows that $S \geq C \geq 0$.

Thus, $G = S - C \geq 0$.

Expected utilities of the complex actions:

- $EU(b \,\&\, A) = EU(A) + G$
- $EU(\neg b \,\&\, A) = EU(A)$
- $EU(b \,\&\, B) = EU(B) - C$
- $EU(\neg b \,\&\, B) = EU(B)$

At least one of $b \,\&\, A$ and $\neg b \,\&\, B$ is admissible.

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This holds even if the agent's net gain is 0 (i.e., $G = S - C = 0$).

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This holds even if the agent's net gain is 0 (i.e., $G = S - C = 0$).

But then it follows that the agent should be willing to accept the bet on A even if $S = C$. Thus, the (fair) betting rate x for A must equal 1 ($P(A) = 1$), Which implies, on pain of incoherence, that $P(B) = 1 - P(A) = 0$. The inadmissible option has probability zero.

Do we have to conclude that probabilities for ones current options must lack any connection at all to ones potential betting behavior?

Rabinowicz: Suppose that the agent is offered an opportunity to make a *betting commitment* with respect to A at stake S and price C. The agent makes a commitment (to buy or sell) not knowing whether he will be required to sell or to buy the bet.

A betting commitment is *fair* if the agent is willing to accept the commitment even if he is *radically uncertain* about what will be required of him.

Game Plan

Today: Towards a model of deliberation in games, equilibrium refinement program, general comments about reasoning and deliberation in game and decision theory

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Tomorrow: Skyrms' dynamic model of Bayesian deliberators.