

Social Choice Theory for Logicians

Lecture 1

Eric Pacuit

Department of Philosophy
University of Maryland, College Park
ai.stanford.edu/~epacuit
epacuit@umd.edu

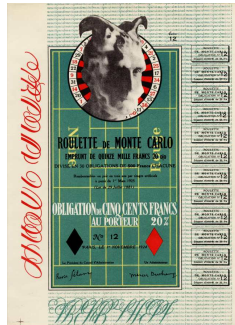
June 18, 2012

`ai.stanford.edu/~epacuit/classes/nasslli2012/logsoc.html`

Context of Decision Making

Context of Decision Making

- ▶ Individual decision making and individual action **against nature**.
 - Example: gambling.



Context of Decision Making

- ▶ Individual decision making and individual action against nature.
- ▶ Individual decision making in **interaction**.
 - Example: playing chess.



Context of Decision Making

- ▶ Individual decision making and individual action against nature.
- ▶ Individual decision making in interaction.
- ▶ **Collective** decision making.
 - Example: carrying a piano, **voting**



Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the “will of the group”?

Main Question

Given a group of people faced with some decision, how should a central authority combine the individual opinions so as to best reflect the “will of the group”?

Typical Examples:

- ▶ Electing government officials
- ▶ Department meetings
- ▶ Deciding where to go to dinner with friends
- ▶

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↓	C	B	C	D
worst	D	D	A	A

Brams and Fishburn. *Voting Procedures*. Handbook of Social Choice and Welfare (2002).

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank A first than any other candidate

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ But, a stronger majority ranks *A* last

Which candidate *should* be chosen?



Marquis de Condorcet (1743 - 1794)

VS.



Jean-Charles de Borda (1733 -1799)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
	B	C	D	B
	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)
- ▶ *B* and *C* are the only candidates not ranked last by anyone

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first (last) than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)
- ▶ Taking into account the *entire* ordering, *B* has the most “support” (*B* is the **Borda winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
3	A	A	B	C
2	B	C	D	B
1	C	B	C	D
0	D	D	A	A

A few observations:

- ▶ More people rank *A* first (last) than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)
- ▶ *B* gets 13 (vs. *A*) + 10 (vs. *C*) + 21 (vs. *D*) = 44 points

Which candidate *should* be chosen?

# voters	3	5	7	6
3	A	A	B	C
2	B	C	D	B
1	C	B	C	D
0	D	D	A	A

A few observations:

- ▶ More people rank *A* first (last) than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)
- ▶ *B* gets 13 (vs. *A*) + 10 (vs. *C*) + 21 (vs. *D*) = 44 points
C gets 13 (vs. *A*) + 11 (vs. *B*) + 14 (vs. *D*) = 38 points

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

A few observations:

- ▶ More people rank *A* first (last) than any other candidate
- ▶ In pairwise elections, *C* beats every other candidate (*C* is the **Condorcet winner**)
- ▶ Taking into account the *entire* ordering, *B* has the most “support” (*B* is the **Borda winner**)

Which candidate *should* be chosen?

# voters	3	5	7	6
best	A	A	B	C
↑	B	C	D	B
↑	C	B	C	D
worst	D	D	A	A

Conclusion: *many ways to answer the above question!*

Choosing How to Choose

Choosing How to Choose

Plurality, Borda Count, Antiplurality/Veto, and k-approval;
Plurality with Runoff; Single Transferable Vote (STV)/Hare;
Approval Voting; Condorcet-consistent methods based on the
simple majority graph (e.g., Cup Rule/Voting Trees, Copeland,
Banks, Slater, Schwartz, and the basic Condorcet rule itself), rules
based on the weighted majority graph (e.g., Maximin/Simpson,
Kemeny, and Ranked Pairs/Tideman), rules requiring full
preference information (e.g., Bucklin, Dodgson, and Young);
Majoritarian Judgment; Cumulative Voting; Range Voting

S.J. Brams and P.C. Fishburn. *Voting Procedures*. In K.J. Arrow et al. (eds.), *Handbook of Social Choice and Welfare*, Elsevier, 2002.

Choosing How to Choose

Plurality, **Borda Count**, Antiplurality/Veto, and k-approval;
Plurality with Runoff; Single Transferable Vote (STV)/Hare;
Approval Voting; Condorcet-consistent methods based on the simple majority graph (e.g., Cup Rule/Voting Trees, Copeland, Banks, Slater, Schwartz, and the basic Condorcet rule itself), rules based on the weighted majority graph (e.g., Maximin/Simpson, Kemeny, and Ranked Pairs/Tideman), rules requiring full preference information (e.g., Bucklin, Dodgson, and Young); Majoritarian Judgment; Cumulative Voting; Range Voting

S.J. Brams and P.C. Fishburn. *Voting Procedures*. In K.J. Arrow et al. (eds.), *Handbook of Social Choice and Welfare*, Elsevier, 2002.

Choosing How to Choose

Plurality Vote: Each voter selects one candidate (or none if voters can abstain) and the candidate(s) with the most votes win.

Plurality with Runoff: If there is a candidate with an absolute majority then that candidate wins, otherwise the top two candidates move on to round two. The candidate with the most votes in the second round wins.

Choosing How to Choose

Approval Voting: Each voter selects a *subset* of the candidates (empty set means the voter abstains) and the candidate(s) with the most votes win.

Borda Count: Each voter provides a linear ordering of the candidates. The candidate(s) with the most total **points** wins, where points are calculated as follows: if there are n candidates, $n - 1$ points are given to the highest ranked candidates, $n - 2$ to the second highest, etc..

How should we judge different social decision methods?

How should we judge different social decision methods?

- ▶ **Pragmatic concerns:** Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.

How should we judge different social decision methods?

- ▶ **Pragmatic concerns:** Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.
- ▶ **Behavioral considerations:** Do the different procedures *really* lead to different outcomes in practice?

How should we judge different social decision methods?

- ▶ **Pragmatic concerns:** Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.
- ▶ **Behavioral considerations:** Do the different procedures *really* lead to different outcomes in practice?
- ▶ **Information required from the voters:** What type of information do the ballots convey? Eg., Choosing a single alternative, linearly rank all the alternatives, report something about the “intensity” of preference.

How should we judge different social decision methods?

- ▶ **Pragmatic concerns:** Is the procedure easy to use? Is it legal? The importance of ease of use should not be underestimated: Despite its many flaws, plurality rule is, by far, the most commonly used method.
- ▶ **Behavioral considerations:** Do the different procedures *really* lead to different outcomes in practice?
- ▶ **Information required from the voters:** What type of information do the ballots convey? Eg., Choosing a single alternative, linearly rank all the alternatives, report something about the “intensity” of preference.
- ▶ **Axiomatics:** Characterize the different social decision methods in terms of normative principles of group decision making.

What properties do we want?

What properties do we want?

- ▶ **Condorcet Candidate:** Always choose the candidate that beats every other candidate in head-to-head elections

What properties do we want?

- ▶ **Condorcet Candidate:** Always choose the candidate that beats every other candidate in head-to-head elections
- ▶ **Monotonicity** A candidate receiving more support shouldn't make her worse off

What properties do we want?

- ▶ **Condorcet Candidate:** Always choose the candidate that beats every other candidate in head-to-head elections
- ▶ **Monotonicity** A candidate receiving more support shouldn't make her worse off
- ▶ **Independence:** The winner should not depend on “irrelevant” spoiler candidates

What properties do we want?

- ▶ **Condorcet Candidate:** Always choose the candidate that beats every other candidate in head-to-head elections
- ▶ **Monotonicity** A candidate receiving more support shouldn't make her worse off
- ▶ **Independence:** The winner should not depend on “irrelevant” spoiler candidates
- ▶ The outcome of a vote should not be “surprising” given the data

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- ▶ Does the group prefer *A* over *B*?

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- ▶ Does the group prefer *A* over *B*? **Yes**

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- ▶ Does the group prefer *A* over *B*? Yes
- ▶ Does the group prefer *B* over *C*? Yes

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- ▶ Does the group prefer *A* over *B*? Yes
- ▶ Does the group prefer *B* over *C*? Yes
- ▶ Does the group prefer *A* over *C*? No

Condorcet Paradox

Voter 1	Voter 2	Voter 3
A	C	B
B	A	C
C	B	A

- ▶ Does the group prefer *A* over *B*? Yes
- ▶ Does the group prefer *B* over *C*? Yes
- ▶ Does the group prefer *A* over *C*? No
(this conflicts with **transitivity**)

W. Gehrlein. *Condorcet's Paradox*. Springer, 2006.

Failure of monotonicity: plurality with runoff

<u># voters</u>	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

<u># voters</u>	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

<u># voters</u>	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

<u># voters</u>	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

Winner: A

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

Winner: A

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
	A	C	B	B
	B	A	C	A
	C	B	A	C

Winner: A

# voters	6	5	4	2
	A	C	B	A
	B	A	C	B
	C	B	A	C

Winner: C

Failure of monotonicity: plurality with runoff

# voters	6	5	4	2
A	C	B	B	
B	A	C	A	
C	B	A	C	

Winner: A

# voters	6	5	4	2
A	C	B	A	
B	A	C	B	
C	B	A	C	

Winner: C

No-show paradox

Totals	Rankings	H over W	W over H
417	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1608		917	691

Fishburn and Brams. *Paradoxes of Preferential Voting*. Mathematics Magazine (1983).

No-show paradox

Totals	Rankings	H over W	W over H
417	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1608		917	691

$$B: 417 + 82 = 499$$

$$H: 143 + 357 = 500$$

$$W: 285 + 324 = 609$$

No-show paradox

Totals	Rankings	H over W	W over H
417	X H W	417	0
82	X W H	0	82
143	H X W	143	0
357	H W X	357	0
285	W X H	0	285
324	W H X	0	324
1608		917	691

H Wins

No-show paradox

Totals	Rankings	H over W	W over H
419	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1610		917	691

Suppose two more people show up with the ranking B H W

No-show paradox

Totals	Rankings	H over W	W over H
419	B H W	417	0
82	B W H	0	82
143	H B W	143	0
357	H W B	357	0
285	W B H	0	285
324	W H B	0	324
1610		917	691

$$B: 419 + 82 = 501$$

$$H: 143 + 357 = 500$$

$$W: 285 + 324 = 609$$

No-show paradox

Totals	Rankings	B over W	W over B
419	B X W	419	0
82	B W X	82	0
143	X B W	143	0
357	X W B	0	357
285	W B X	0	285
324	W X B	0	324
1610		644	966

$$B: 419 + 82 = 501$$

$$H: 143 + 357 = 500$$

$$W: 285 + 324 = 609$$

No-show paradox

Totals	Rankings	B over W	W over B
419	B X W	419	0
82	B W X	82	0
143	X B W	143	0
357	X W B	0	357
285	W B X	0	285
324	W X B	0	324
1610		644	966

W Wins!

Failure of Independence

<u># voters</u>	3	2	2
	A	B	C
	B	C	A
	C	A	B

Failure of Independence

<u># voters</u>	3	2	2
	A	B	C
	B	C	A
	C	A	B

- ▶ The BC ranking is: $A (8) > B (7) > C (6)$

Failure of Independence

# voters	3	2	2
	A	B	C
	B	C	X
	C	X	A
	X	A	B

- ▶ The BC ranking is: $A (8) > B (7) > C (6)$
- ▶ Add a new (undesirable) candidate X

Failure of Independence

# voters	3	2	2
	A	B	C
	B	C	X
	C	X	A
	X	A	B

- ▶ The BC ranking is: $A (8) > B (7) > C (6)$
- ▶ Add a new (undesirable) candidate X
- ▶ The new BC ranking is: $C (13) > B (12) > A (11) > X (6)$

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYY	NYN	NNY	NNN
1	1	1	3	1	3	3	0

S. Brams, D. M. Kilgour, and W. Zwicker. *"The paradox of multiple elections"*. Social Choice and Welfare, 15(2): 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYN	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)

S. Brams, D. M. Kilgour, and W. Zwicker. "The paradox of multiple elections". *Social Choice and Welfare*, 15(2): 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYN	YYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)

Proposition 2: *N* (7 - 6)

S. Brams, D. M. Kilgour, and W. Zwicker. "The paradox of multiple elections". *Social Choice and Welfare*, 15(2): 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYN	NYN	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: *N* (7 - 6)

Proposition 2: *N* (7 - 6)

Proposition 3: *N* (7 - 6)

S. Brams, D. M. Kilgour, and W. Zwicker. "The paradox of multiple elections". *Social Choice and Welfare*, 15(2): 211 - 236, 1998.

Multiple Elections Paradox

Voters are asked to give their opinion on three yes/no issues:

YYY	YYN	YNY	YNN	NYN	NYY	NNY	NNN
1	1	1	3	1	3	3	0

Outcome by majority vote

Proposition 1: N (7 - 6)

Proposition 2: N (7 - 6)

Proposition 3: N (7 - 6)

But there is no support for NNN!

S. Brams, D. M. Kilgour, and W. Zwicker. "The paradox of multiple elections". *Social Choice and Welfare*, 15(2): 211 - 236, 1998.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

Voters 1,2 & 3 do not support the outcome on a majority of issues

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

Anscombe's Paradox

	Issue 1	Issue 2	Issue 3
Voter 1	Yes	Yes	No
Voter 2	No	No	No
Voter 3	No	Yes	Yes
Voter 4	Yes	No	No
Voter 5	Yes	No	Yes
Majority	Yes	No	Yes

Voters 4 & 5 support the outcome on a majority of issues

Voters 1,2 & 3 do not support the outcome on a majority of issues

A majority of voters do not support the majority outcome on a majority of issues.

G. E. M. Anscombe. *On Frustration of the Majority by Fulfillment of the Majority's Will*. *Analysis*, 36(4): 161-168, 1976.

What properties do we want?

- ▶ **Condorcet Candidate:** Always choose the candidate that beats every other candidate in head-to-head elections
- ▶ **Monotonicity** A candidate receiving more support shouldn't make her worse off
- ▶ **Independence:** The winner should not depend on “irrelevant” spoiler candidates
- ▶ The outcome of a vote should not be “surprising” given the data

Arrow's Theorem

K. Arrow. *Social Choice & Individual Values*. 1951.

Also, see

J. Geanakoplos. *Three Brief Proofs of Arrow's Impossibility Theorem*. *Economic Theory*, **26**, 2005.

A. Taylor. *Social Choice and The Mathematics of Manipulation*. Cambridge University Press, 2005.

W. Gaertner. *A Primer in Social Choice Theory*. Oxford University Press, 2006.

Sen's Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

Sen's Liberal Paradox

Two members of a small society Lewd and Prude each have a personal copy of *Lady Chatterley's Lover*, consider

l : Lewd reads the book;

p : Prude reads the book;

$l \rightarrow p$: If Lewd reads the book, then so does Prude.

Sen's Liberal Paradox

Sen's Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

Sen's Liberal Paradox

Lewd desires to read the book, and if he reads it, then so does Prude (Lewd enjoys the thought of Prude's moral outlook being corrupted)

Prude desires to not read the book, and that Lewd not read it either, but in case Lewd does read the book, Prude wants to read the book to be informed about the dangerous material Lewd has read.

Sen's Liberal Paradox

	l	p	$l \rightarrow p$

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
 l is Lewd's case, p is Prude's case

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
 l is Lewd's case, p is Prude's case
2. Unanimous desires of all individuals must be respected.

Sen's Liberal Paradox

	l	p	$l \rightarrow p$
Lewd	True	True	True
Prude	False	False	True

1. Society assigns to each individual the liberal right to determine the collective desire on those propositions that concern only the individual's private sphere
 l is Lewd's case, p is Prude's case
2. Unanimous desires of all individuals must be respected.

So, society must be inconsistent!

Muller-Satterthwaite Theorem

E. Muller and M. A. Satterthwaite. *The equivalence of strong positive association and strategy-proofness*. Journal of Economic Theory, 14(2):412-418, 1977.

P. Tang and T. Sandholm. *Coalitional Structure of the Muller-Satterthwaite Theorem*. In *Proceedings of the Workshop on Cooperative Games in Multiagent Systems (CoopMAS)* at AAMAS, 2012.