Problem Set # 2

1. Show that $(\Box\Diamond p \wedge \Diamond\Box(p \to q)) \to \Diamond\Diamond q$ is valid using the rules for sequents described in Section 4.3 of *Modal Logic for Open Minds* (see also my notes on sequents available on the course website)

2. Find a derivation of $\Box\Diamond\Box\Diamond p \leftrightarrow \Box\Diamond p$ in **K4** (the normal modal logic plus the axiom $\Box p \to \Box\Box p$). *See the hint in the solutions for the exercises in Chapter 5 in the textbook.*

3. A frame $\langle W, R\rangle$ is secondary reflexive provide for each w, v if wRv then vRv.

   (a) Prove that $\mathcal{F} = \langle W, R\rangle$ is secondary reflexive iff $\mathcal{F} \models \Box(\Box\varphi \to \varphi)$

   (b) Prove that $\Box(\Box\varphi \to \varphi)$ is canonical for secondary reflexivity.

4. Consider the axiom $\Diamond p \to \Box p$. Let **KP** be the normal modal logic containing **K** plus this axiom. Prove that **KP** is sound and strongly complete with respect to the class of all frames $\langle W, R\rangle$ where R is a partial function (i.e., R satisfies the property for all w, v_1, v_2, if wRv_1 and wRv_2 then $v_1 = v_2$).

5. (Extra Credit) Given a class K of frames, let $\Theta(\mathsf{K}) = \Lambda_{\mathsf{K}}$ denote the set $\{\varphi \mid \mathcal{F} \models \varphi \text{ for all } \mathcal{F} \in \mathsf{K}\}$ and given a logic $\mathbf{L}$, let $\mathsf{Fr}(\mathbf{L})$ denote the class of frames on which $\mathbf{L}$ is valid.

   (a) Show that Θ and Fr form a *Galois connection*. That is prove that for all classes K and logic $\mathbf{L}$:
   $$\mathbf{L} \subseteq \Theta(\mathsf{K}) \text{ iff } \mathsf{K} \subseteq \mathsf{Fr}(\mathbf{L})$$

   (b) What does it mean for a logic $\mathbf{L}$ if $\mathbf{L} = \Theta(\mathsf{Fr}(\mathbf{L}))$? (Give an example of a logic for which it does not hold.)

   (c) What does it mean for a frame class K if $\mathsf{K} = \mathsf{Fr}(\Theta(\mathsf{K}))$? (Give an example of a frame class for which it does not hold).

The homework is DUE Tuesday, March 27, 2012.