Logics of Rational Agency

Lecture 3

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July 29, 2009

✓ Introduction, Motivation and Background

Lecture 2: Basic Ingredients for a Logic of Rational Agency

Lecture 3: Logics of Rational Agency and Social Interaction, Part I

Lecture 4: Logics of Rational Agency and Social Interaction, Part II

Lecture 5: Conclusions and General Issues

Logics of Rational Agency

Basic Ingredients

- ✓ informational attitudes (eg., knowledge, belief, certainty)
- ✓ group notions (eg., common knowledge and coalitional ability)
- ✓ time, actions and ability
- ✓ motivational attitudes (eg., preferences)
- ✓ normative attitudes (eg., obligations)

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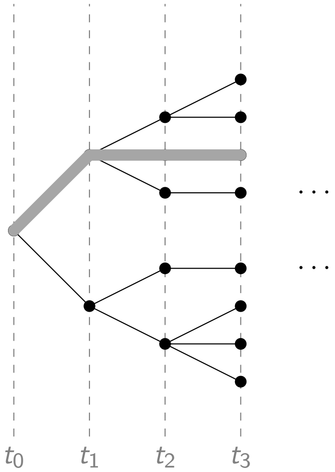
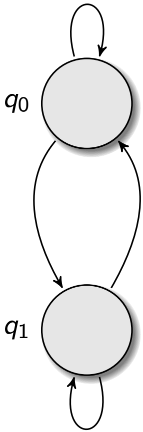
Temporal Logics



Computational vs. Behavioral Structures



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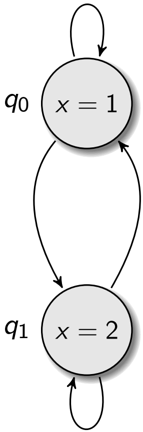

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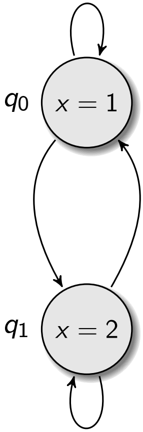

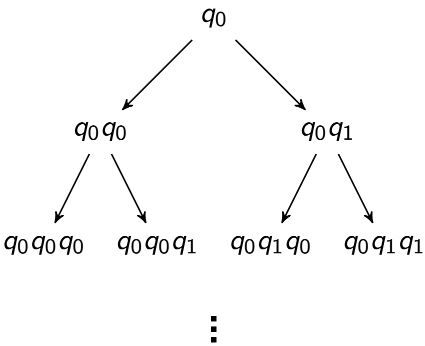

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- *Linear Time Temporal Logic*: Reasoning about computation paths:

$$\Diamond\varphi: \varphi \text{ is true some time in } \textit{the} \text{ future.}$$

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- *Branching Time Temporal Logic*: Allows quantification over paths:

$$\exists\Diamond\varphi\text{: there is a path in which } \varphi \text{ is eventually true.}$$

E. M. Clarke and E. A. Emerson. *Design and Synthesis of Synchronization Skeletons using Branching-time Temproal-logic Specifications*. In *Proceedings Workshop on Logic of Programs*, LNCS (1981).

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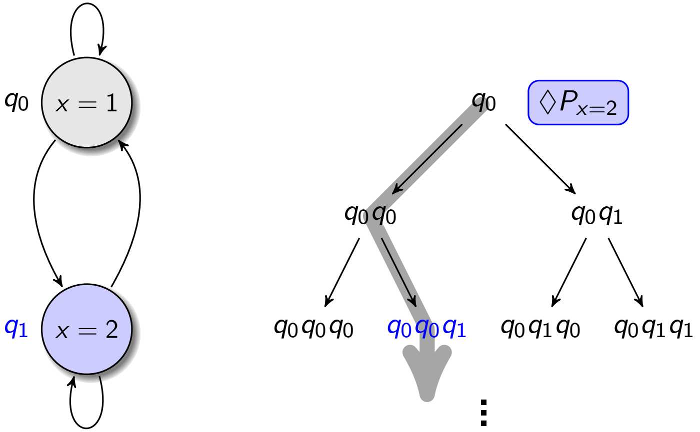

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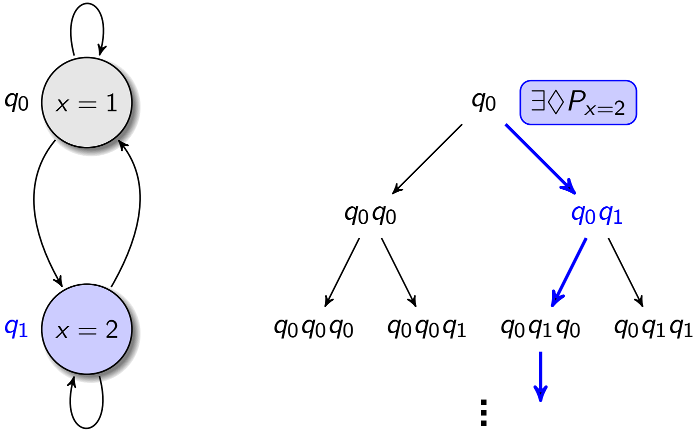

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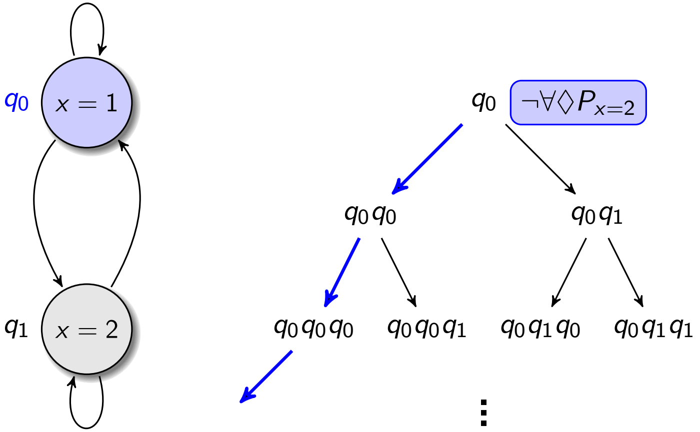

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From Temporal Logic to Strategy Logic

- *Coalitional Logic*: Reasoning about (local) group power.

$[C]\varphi$: coalition C has a **joint action** to bring about φ.

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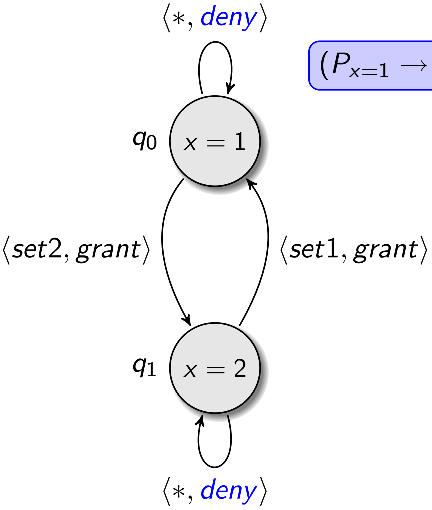
- *Alternating-time Temporal Logic*: Reasoning about (local and global) group power:

  $\langle\langle A\rangle\rangle\Box\varphi$: The coalition A has a **joint action** to ensure that φ will remain true.

R. Alur, T. Henzinger and O. Kupferman. *Alternating-time Temproal Logic*. *Jouranl of the ACM* (2002).

Multi-agent Transition Systems



$(P_{x=1} \rightarrow [s]P_{x=1}) \wedge (P_{x=2} \rightarrow [s]P_{x=2})$

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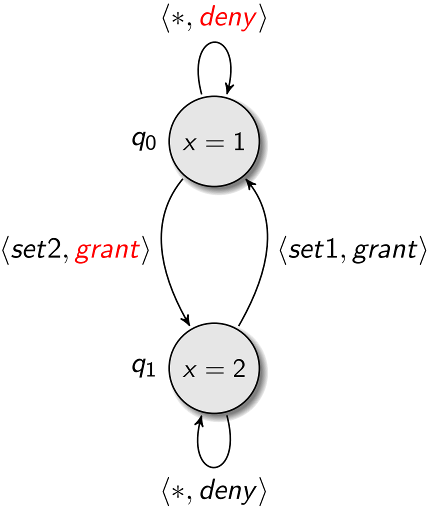

$$P_{x=1} \rightarrow \neg[s]P_{x=2}$$

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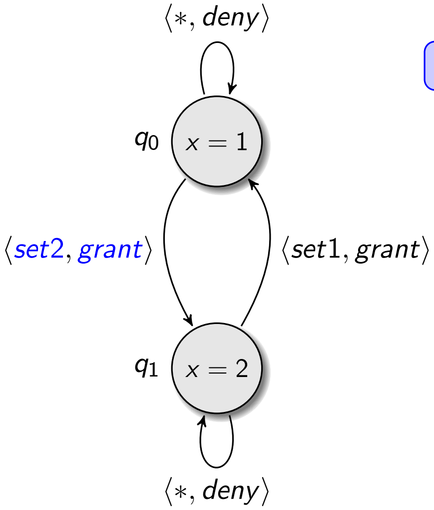

$P_{x=1} \longrightarrow \neg[s, c]P_{x=2}$

Preference (Modal) Logics

x, y objects

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1. $x \succeq y$ and $y \not\succeq x$ ($x \succ y$)
2. $x \not\succeq y$ and $y \succeq x$ ($y \succ x$)
3. $x \succeq y$ and $y \succeq x$ ($x \sim y$)
4. $x \not\succeq y$ and $y \not\succeq x$ ($x \perp y$)

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Properties: transitivity, connectedness, etc.

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Preference Modalities $\langle \succeq \rangle \varphi$: "there is a world at least as good (as the current world) satisfying φ"

$\mathcal{M}, w \models \langle \succeq \rangle \varphi$ iff there is a $v \succeq w$ such that $\mathcal{M}, v \models \varphi$

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$\mathcal{M}, w \models \langle \succ \rangle \varphi$ iff there is $v \succeq w$ and $w \not\succeq v$ such that $\mathcal{M}, v \models \varphi$

Preference (Modal) Logics

1. $\langle \succ \rangle \varphi \rightarrow \langle \succeq \rangle \varphi$
2. $\langle \succeq \rangle \langle \succ \rangle \varphi \rightarrow \langle \succ \rangle \varphi$
3. $\varphi \wedge \langle \succeq \rangle \psi \rightarrow (\langle \succ \rangle \psi \vee \langle \succeq \rangle (\psi \wedge \langle \succeq \rangle \varphi))$
4. $\langle \succ \rangle \langle \succeq \rangle \varphi \rightarrow \langle \succ \rangle \varphi$

Theorem The above logic (with Necessitation and Modus Ponens) is sound and complete with respect to the class of preference models.

J. van Benthem, O. Roy and P. Girard. *Everything else being equal: A modal logic approach to* ceteris paribus *preferences*. JPL, 2008.

Preference Modalities

$\varphi \geq \psi$: the state of affairs φ is at least as good as ψ (ceteris paribus)

G. von Wright. *The logic of preference*. Edinburgh University Press (1963).

From worlds to sets and back

Lifting

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Deriving
$P_1 \gg P_2 \gg P_3 \gg \cdots \gg P_n$
$x > y$ iff x and y differ in at least one P_i and the first P_i where this happens is one with $P_i x$ and $\neg P_i y$

F. Liu and D. De Jongh. *Optimality, belief and preference*. 2006.

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Lecture 5: Conclusions and General Issues

General Issues

Once a semantics and language are fixed, then standard questions can be asked: eg. develop a proof theory, completeness, decidability, model checking.

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- Comparing different frameworks: eg. PDL vs. Temporal Logic, PDL vs. STIT, STIT vs. ATL, etc.

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Theorem $\Box\varphi \leftrightarrow \varphi$ is provable in combinations of Epistemic Logics and PDL with certain "cross axioms" ($\Box[a]\varphi \leftrightarrow [a]\Box\varphi$) (and full substitution).

R. Schmidt and D. Tishkovsky. *On combinations of propositional dynamic logic and doxastic modal logics*. JOLLI, 2008.

Merging logics of rational agency

- Reasoning about information change (knowledge and time/actions)
- Knowledge, beliefs and certainty
- "Epistemizing" logics of action and ability: *knowing how to achieve* φ vs. *knowing that you can achieve* φ
- Entangling knowledge and preferences
- Planning/intentions (BDI)

Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

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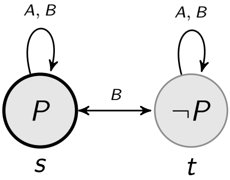
Is this procedure correct? Yes, if

1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

Example



P means "The talk is at 2PM".

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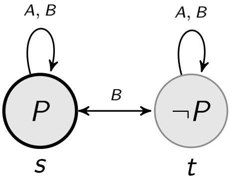

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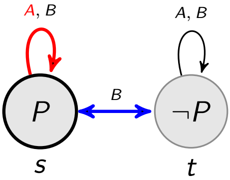

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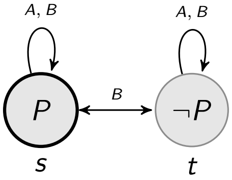
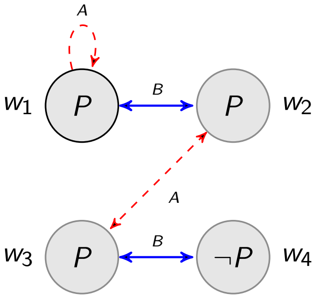


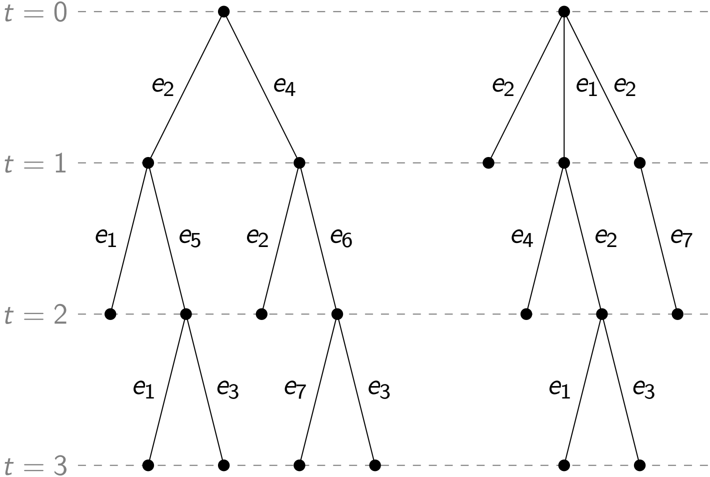


Epistemic Temporal Logic

R. Parikh and R. Ramanujam. *A Knowledge Based Semantics of Messages*. *Journal of Logic, Language and Information*, 12: 453 – 467, 1985, 2003.

FHMV. Reasoning about Knowledge. MIT Press, 1995.

The ‘Playground’



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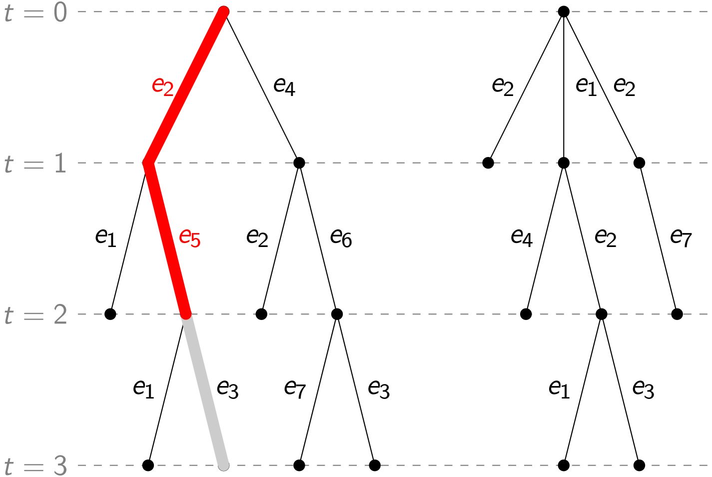

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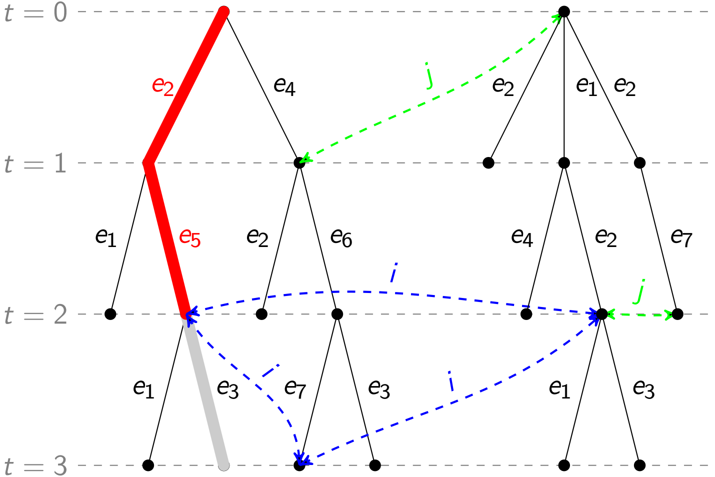

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- ϵ is the empty string and $\mathsf{FinPre}_{-\epsilon}(\mathcal{H}) = \mathsf{FinPre}(\mathcal{H}) - \{\epsilon\}$.

History-based Frames

Definition

Let Σ be any set of events. A set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ is called a protocol provided $\mathsf{FinPre}_{-\epsilon}(\mathcal{H}) \subseteq \mathcal{H}$. A rooted protocol is any set $\mathcal{H} \subseteq \Sigma^* \cup \Sigma^\omega$ where $\mathsf{FinPre}(\mathcal{H}) \subseteq \mathcal{H}$.

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Definition

An ETL frame is a tuple $\langle \Sigma, \mathcal{H}, \{\sim_i\}_{i \in \mathcal{A}} \rangle$ where Σ is a (finite or infinite) set of events, $\mathcal{H}$ is a protocol, and for each $i \in \mathcal{A}$, $\sim_i$ is an equivalence relation on the set of finite strings in $\mathcal{H}$.

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Some assumptions:

1. If Σ is assumed to be finite, then we say that $\mathcal{F}$ is **finitely branching**.
2. If $\mathcal{H}$ is a rooted protocol, $\mathcal{F}$ is a **tree frame**.

Formal Languages

- $P\varphi$ (φ is true *sometime* in the past),
- $F\varphi$ (φ is true *sometime* in the future),
- $Y\varphi$ (φ is true at *the* previous moment),
- $N\varphi$ (φ is true at *the* next moment),
- $N_e\varphi$ (φ is true after event e)
- $K_i\varphi$ (agent i knows φ) and
- $C_B\varphi$ (the group $B \subseteq \mathcal{A}$ commonly knows φ).

History-based Models

An ETL **model** is a structure $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}, V\rangle$ where $\langle \mathcal{H}, \{\sim_i\}_{i\in\mathcal{A}}\rangle$ is an ETL frame and

$V : \mathsf{At} \rightarrow 2^{\mathsf{finite}(\mathcal{H})}$ is a valuation function.

Formulas are interpreted at pairs H, t:

$$H, t \models \varphi$$

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
- $H, t \models C\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_* H'_m$ then $H', m \models \varphi$.

where $\sim_*$ is the reflexive transitive closure of the union of the $\sim_i$.

Truth in a Model

- $H, t \models P\varphi$ iff there exists $t' \leq t$ such that $H, t' \models \varphi$
- $H, t \models F\varphi$ iff there exists $t' \geq t$ such that $H, t' \models \varphi$
- $H, t \models N\varphi$ iff $H, t+1 \models \varphi$
- $H, t \models Y\varphi$ iff $t > 1$ and $H, t-1 \models \varphi$
- $H, t \models K_i\varphi$ iff for each $H' \in \mathcal{H}$ and $m \geq 0$ if $H_t \sim_i H'_m$ then $H', m \models \varphi$
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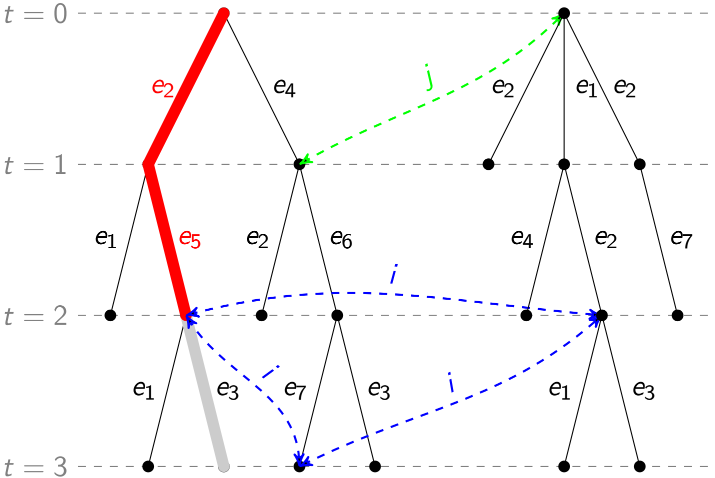

$t = 0$
$t = 1$
$t = 2$
$t = 3$
e_2
e_4
j
e_2
e_1
e_2
e_1
e_5
e_2
e_6
e_4
e_2
e_7
i
e_1
e_3
i
e_7
e_3
i
e_1
e_3

Returning to the Example

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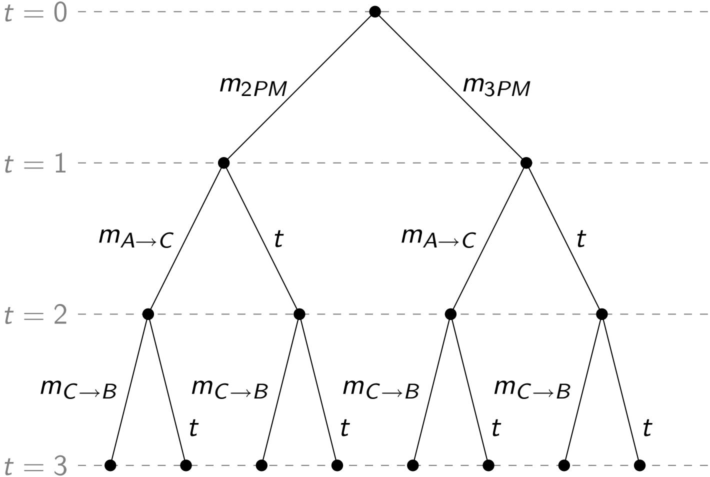
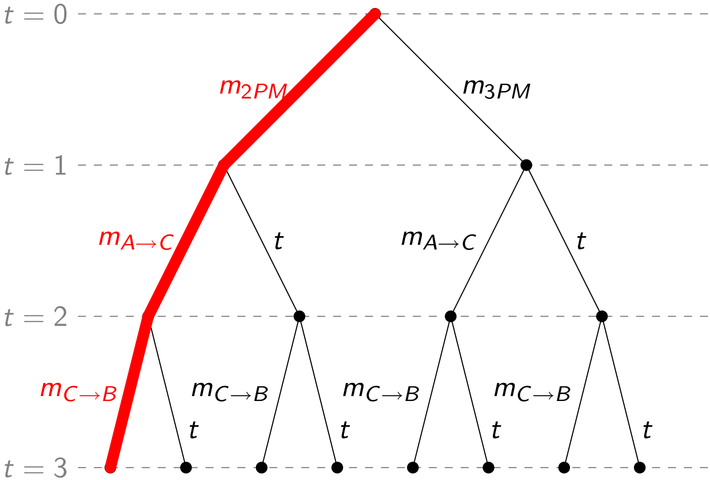
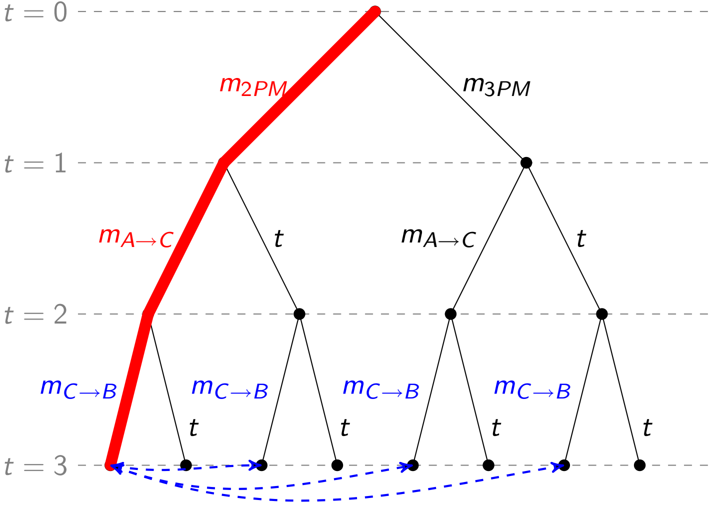
Is this procedure correct?



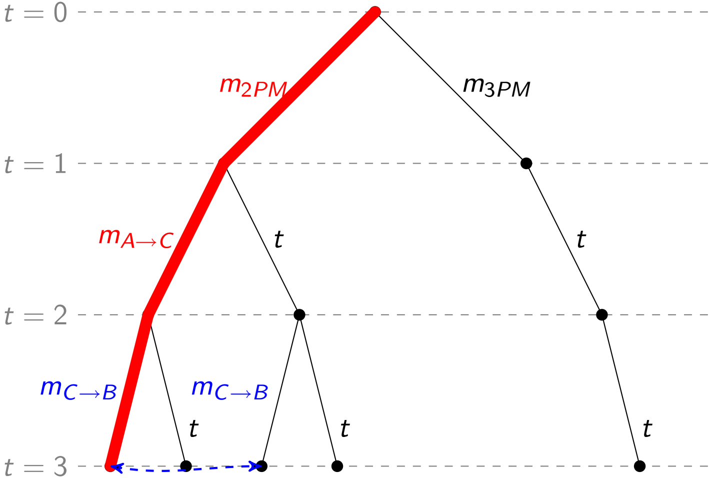
$t = 0$
$t = 1$
$t = 2$
$t = 3$
m_{2PM}
m_{3PM}
$m_{A \to C}$
t
$m_{A \to C}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t
$m_{C \to B}$
t



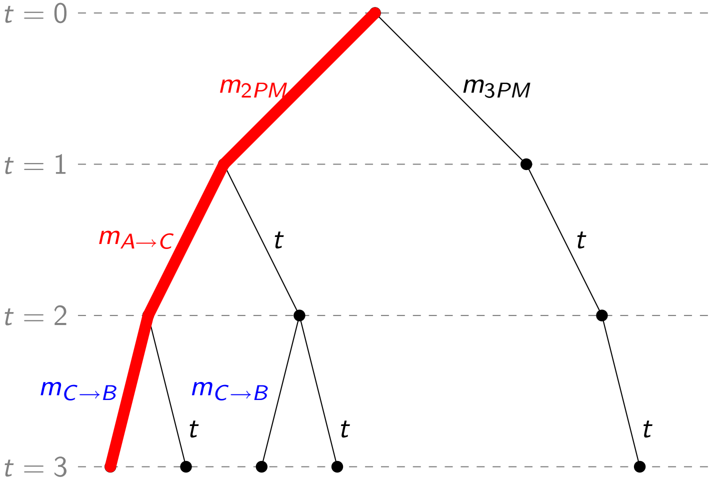
$$H, 3 \models \varphi$$



Bob's uncertainty: $H, 3 \models \neg K_B P_{2PM}$



Bob's uncertainty + 'Protocol information': $H, 3 \models K_B P_{2PM}$



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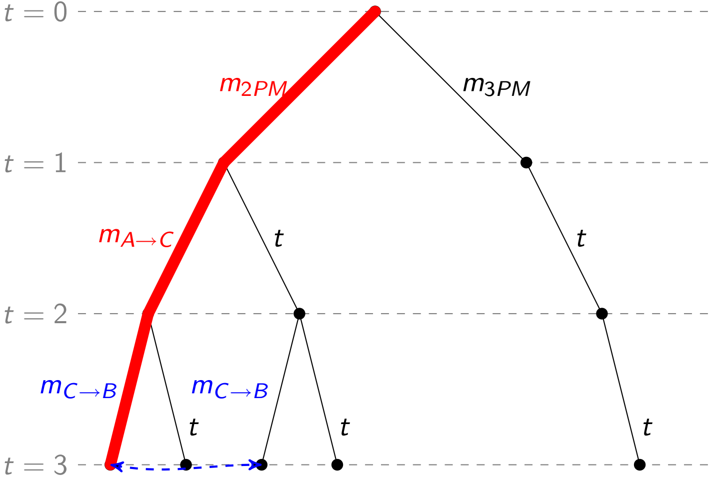

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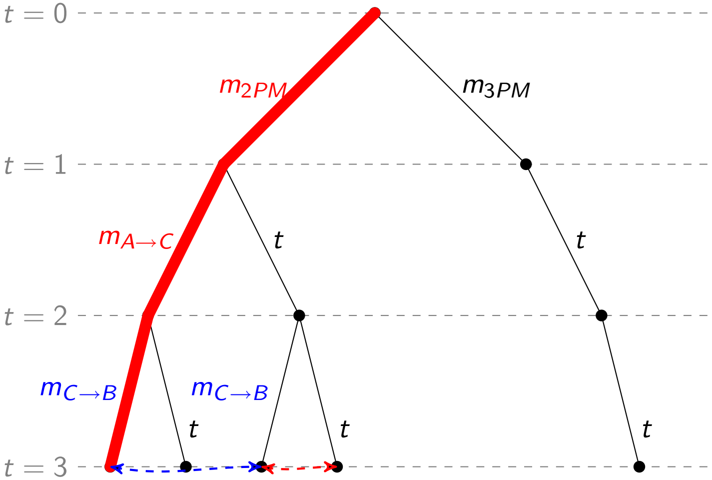

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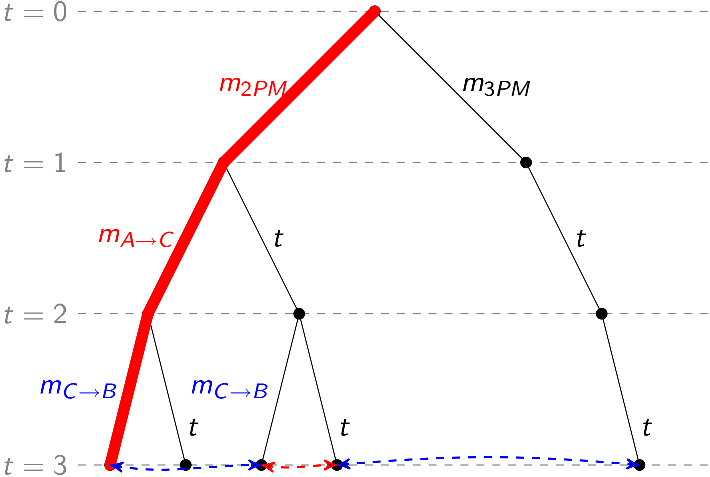

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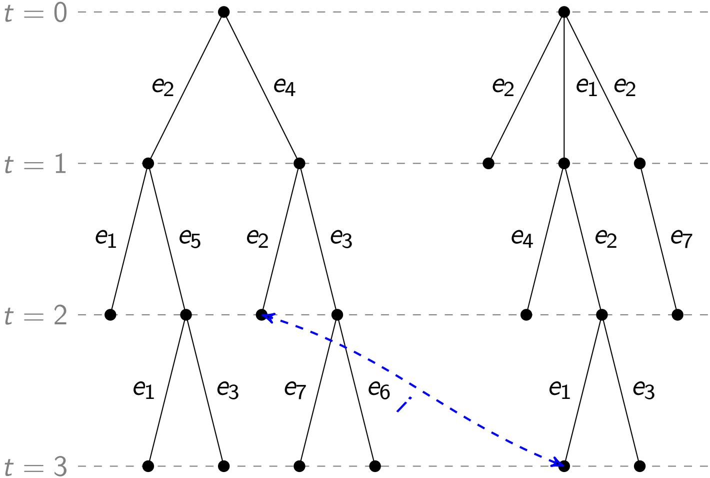
3. Conditions on the reasoning abilities of the agents. Do the agents satisfy perfect recall? No miracles? Do they agents' know what time it is?

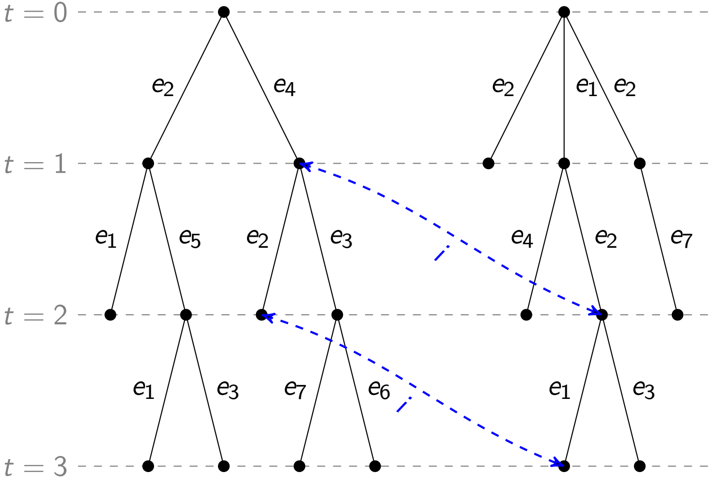
Agent Oriented Properties:

- **No Miracles**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $H \sim_i H'$ then $He \sim_i H'e$.
- **Perfect Recall**: For all finite histories $H, H' \in \mathcal{H}$ and events $e \in \Sigma$ such that $He \in \mathcal{H}$ and $H'e \in \mathcal{H}$, if $He \sim_i H'e$ then $H \sim_i H'$.
- **Synchronous**: For all finite histories $H, H' \in \mathcal{H}$, if $H \sim_i H'$ then $\text{len}(H) = \text{len}(H')$.

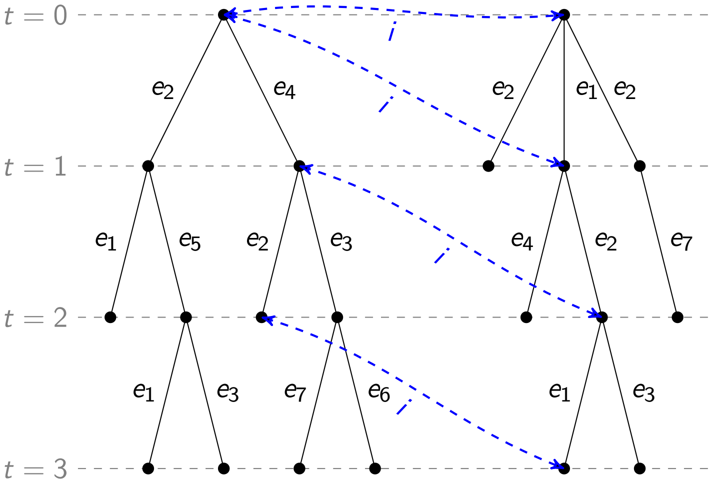
Perfect Recall



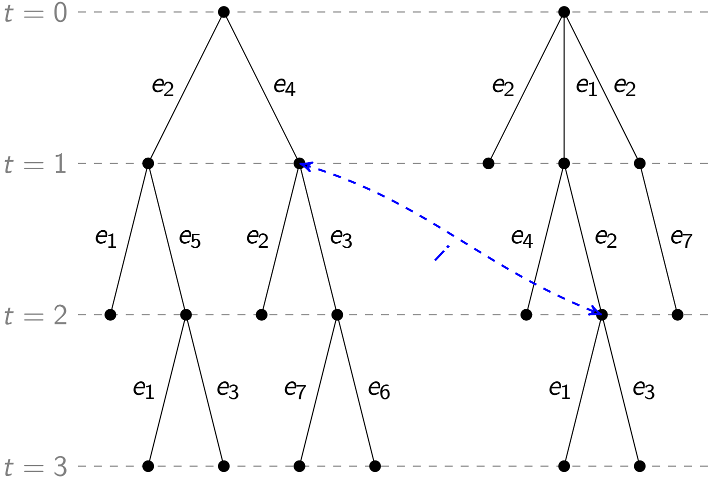
Perfect Recall



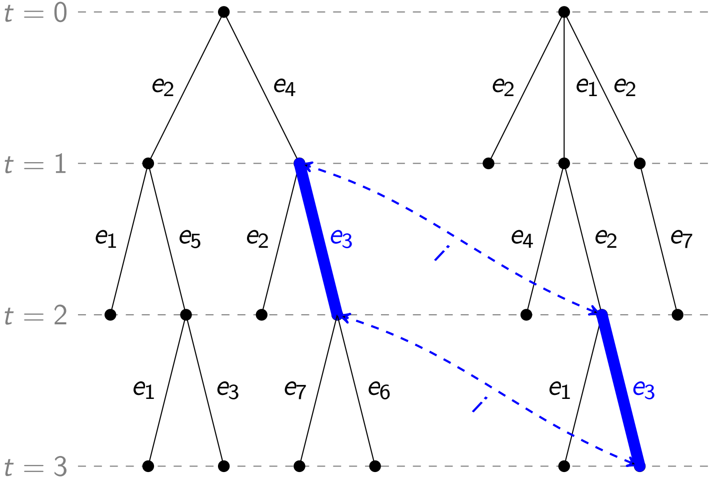
Perfect Recall



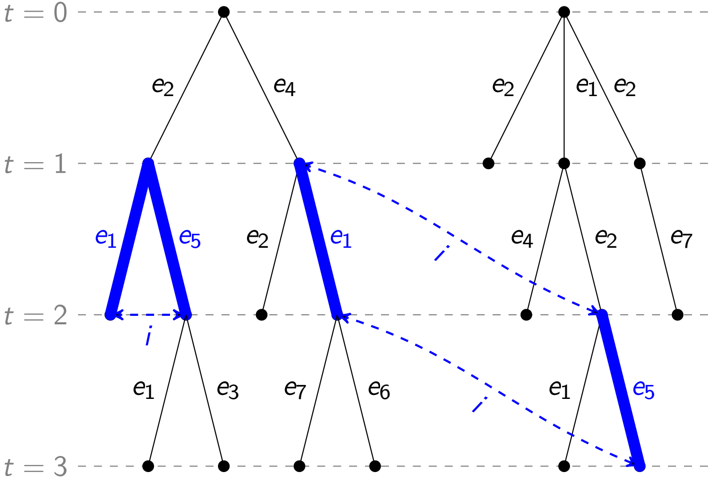
No Miracles



No Miracles



No Miracles



Ideal Agents

Assume there are two agents

Theorem

The logic of ideal *agents with respect to a language with common knowledge and future is highly undecidable (for example, by assuming perfect recall).*

J. Halpern and M. Vardi.. *The Complexity of Reasoning abut Knowledge and Time*. *J. Computer and Systems Sciences*, 38, 1989.

J. van Benthem and EP. *The Tree of Knowledge in Action*. Proceedings of AiML, 2006.

End of lecture 3.