Introduction to Formal Epistemology

Lecture 5

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✓ Introduction, Motivation and Basic Epistemic Logic

✓ Other models of Knowledge, Knowledge in Groups and Group Knowledge

✓ Adding Dynamics, Reasoning about Knowledge in Games

✓ Logical Omniscience and Other Problems

Lecture 5: Reasoning about Knowledge in the Context of Social Software

Social Procedures

- Fair Division Algorithms
- Voting Procedures

Adjusted Winner

Adjusted winner (AW) is an algorithm for dividing n divisible goods among two people (invented by Steven Brams and Alan Taylor).

For more information see

- *Fair Division: From cake-cutting to dispute resolution* by Brams and Taylor, 1998
- *The Win-Win Solution* by Brams and Taylor, 2000
- `www.nyu.edu/projects/adjustedwinner`

Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods: A, B, and C.

Step 1. Both Ann and Bob divide 100 points among the three goods.

Item	Ann	Bob
A	5	4
B	65	46
C	30	50
Total	100	100

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Item	Ann	Bob
A	5	0
B	65	0
C	0	50
Total	70	50

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Step 3. Equitability adjustment:

Notice that $65/46 \geq 5/4 \geq 1 \geq 30/50$

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Give A to Bob (the item whose ratio is closest to 1)

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C	0	50
Total	65	54

Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods: A, B, and C.

Step 3. Equitability adjustment:

Still not equal, so give (some of) B to Bob: $65p = 100 - 46p$.

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A	0	4
B	65	0
C	0	50
Total	65	54

Adjusted Winner: Example

Suppose Ann and Bob are dividing three goods: A, B, and C.

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yielding $p = 100/111 = 0.9009$

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Suppose Ann and Bob are dividing three goods: A, B, and C.

Step 3. Equitability adjustment:

yielding $p = 100/111 = 0.9009$

Item	Ann	Bob
A	0	4
B	58.559	4.559
C	0	50
Total	58.559	58.559

Adjusted Winner: Formal Definition

Suppose that $G_1, \ldots, G_n$ is a fixed set of goods.

A **valuation** of these goods is a vector of natural numbers $\langle a_1, \ldots, a_n \rangle$ whose sum is 100.

Let $\alpha, \alpha', \alpha'', \ldots$ denote possible valuations for Ann and $\beta, \beta', \beta'', \ldots$ denote possible valuations for Bob.

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Suppose that $G_1, \ldots, G_n$ is a fixed set of goods.

An **allocation** is a vector of n real numbers where each component is between 0 and 1 (inclusive). An allocation $\sigma = \langle s_1, \ldots, s_n \rangle$ is interpreted as follows.

For each $i = 1, \ldots, n$, s_i is the proportion of G_i given to Ann.

Thus if there are three goods, then $\langle 1, 0.5, 0 \rangle$ means, "Give all of item 1 and half of item 2 to Ann and all of item 3 and half of item 2 to Bob."

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Fairness

- **Proportional** if both Ann and Bob receive at least 50% of their valuation: $\sum_{i=1}^{n} s_i a_i \geq 50$ and $\sum_{i=1}^{n}(1 - s_i)b_i \geq 50$
- **Envy-Free** if no party is willing to give up its allocation in exchange for the other player's allocation: $\sum_{i=1}^{n} s_1 a_i \geq \sum_{i=1}^{n}(1 - s_i)a_i$ and $\sum_{i=1}^{n}(1 - s_i)b_i \geq \sum_{i=1}^{n} s_i b_i$
- **Equitable** if both players receive the same total number of points: $\sum_{i=1}^{n} s_i a_i = \sum_{i=1}^{n}(1 - s_i)b_i$
- **Efficient** if there is no other allocation that is strictly better for one party without being worse for another party: for each allocation $\sigma' = \langle s'_1, \ldots, s'_n \rangle$ if $\sum_{i=1}^{n} a_i s'_i > \sum_{i=1}^{n} a_i s_i$, then $\sum_{i=1}^{n}(1 - s'_i)b_i < \sum_{i=1}^{n}(1 - s_i)b_i$. (Similarly for Bob)

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Easy Observations

- For two-party disputes, proportionality and envy-freeness are equivalent.
- *AW* only produces equitable allocations (equitability is essentially built in to the procedure).
- *AW* produces allocations σ that in which at most one good is split.

Adjusted Winner is Fair

Theorem (Brams and Taylor) *AW produces allocations that are efficient, equitable and envy-free (with respect to the announced valuations)*

S. Brams and A. Taylor. Fair Division. Cambridge University Press.

Adjusted Winner: Strategizing

Item	Ann	Bob
Matisse	75	25
Picasso	25	75

Ann will get the Matisse and Bob will get the Picasso and each gets 75 of his or her points.

Adjusted Winner: Strategizing

Suppose Ann knows Bob's preferences, but Bob does not know Ann's.

Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	25
P	74	75

So Ann will get M plus a portion of P.

According to Ann's announced allocation, she receives 50 points

According to Ann's actual allocation, she receives $75 + 0.33 * 25 = 83.33$ points.

Strategizing: A Theorem

Theorem (Brams and Taylor) *Assume there are two goods, G_1 and G_2, all true and announced values are restricted to integers, and suppose Bob's announced valuation of G_1 is x, where $x \geq 50$. Assume Ann's true valuation of G_1 is b. Then her optimal announced valuation of G_1 is:*

$$\begin{cases} x+1 & \text{if } b > x \\ x & \text{if } b = x \\ x-1 & \text{if } b < x \end{cases}$$

Strategizing: Example

Suppose *both* players know each other's preferences but neither knows that the other knows their own preference.

Item	Ann	Bob
M	75	25
P	25	75

Item	Ann	Bob
M	26	74
P	74	26

Each will get 74 of his or her announced points, but each one is really getting only 25 of his or her *true* points.

Strategizing: Example

Suppose *both* players know each other's preferences. Moreover, Ann knows that Bob knows her preference and Bob doesn't know that Ann knows.

Item	Ann	Bob
M	26	74
P	74	26

Item	Ann	Bob
M	73	74
P	27	26

What happens as the level of knowledge increases?

- Fair Division
- Voting

The Gibbard-Satterthwaite Theorem

Theorem There must be situations where it 'profits' a voter to vote *strategically*, i.e., not according to his or her *actual preference*.

Under suitable conditions,

1. If P denotes the actual preference ordering of voter i,
2. and $\vec{Y}$ denotes the profile consisting of the preference orderings of all the other voters,
3. and S the aggregation rule,

Then the theorem says that there must exist P, Y, P' such that $S(P', Y) >_P S(P, Y)$.

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Two Issues

1. What does it *mean* to vote strategically?
   - Voting as a game vs. voting as an act of communication

R. Parikh and E. Pacuit. *Safe Votes, Sincere Votes and Strategizing*. presented at Stony Brook Game Theory Conference, 2005.

2. When is the Gibbard-Satterthwaite Theorem '*effective*'?
   - The decision to strategize depends on the agents' *information* (eg. poll information).

E. Pacuit and R. Parikh. *Knowledge Considerations in Strategic Voting*. Working Paper.

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Voting Problem

Given a (finite) set X of **candidates**

and a (finite) set A of **voters**

each of whom have a **preference** over X

Devise a method F which aggregates the individual preferences to produce a collective decision (typically a subset of X)

Voting Procedures

- Type of vote, or **ballot**, that is recognized as admissible by the procedure: let $\mathcal{B}(X)$ be the set of admissible ballots for a set X of candidates
- A method to **count** a vector of ballots (one ballot for each voter) and select a winner (or winners)

Formally, A voting procedure for a set A of agents (with $|A| = n$) and a set X of candidates is a pair

$$(\mathcal{B}(X), \mathrm{Ag})$$

- $\mathcal{B}(X)$ is a set of ballots; and
- $\mathrm{Ag} : \mathcal{B}(X)^n \rightarrow 2^X$ (typically we are interested in the case where $|\mathrm{Ag}(\vec{b})| = 1$).

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Examples

Plurality (Simple Majority)

- $\mathcal{B}(X) = X$
- Given $\vec{b} \in X^n$ and $x \in X$, let $\#_x(\vec{b}) = \sum_{\{i \mid b_i = x\}} 1$

$$\mathrm{Ag}(\vec{b}) = \{x \mid \#_x(\vec{b}) \text{ is maximal}\}$$

Approval Voting

- $\mathcal{B}(X) = 2^X$
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Strategizing Functions

Fix the voters' true preferences: $\mathcal{P}^* = (P_1^*, \ldots, P_n^*)$

Given a vote profile $\vec{v}$ of *actual* votes, we ask whether voter i will change its vote if given another chance to vote.

Example I

The following example is due to [BF]

$$P_A^* = o_1 > o_3 > o_2$$
$$P_B^* = o_2 > o_3 > o_1$$
$$P_C^* = o_3 > o_1 > o_2$$

Size	Group	I	II
4	A	$\mathbf{o_1}$	$\mathbf{o_1}$
3	B	o_2	o_2
2	C	o_3	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. o' is one of the top two candidates as indicated by a poll
2. o' is preferred to the other top candidate

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Example II

$P_A^* = (o_1, o_4, o_2, o_3)$

$P_B^* = (o_2, o_1, o_3, o_4)$

$P_C^* = (o_3, o_2, o_4, o_1)$

$P_D^* = (o_4, o_1, o_2, o_3)$

$P_E^* = (o_3, o_1, o_2, o_4)$

Size	Group	I	II	III	IV
40	A	$\mathbf{o_1}$	o_1	o_4	$\mathbf{o_1}$
30	B	o_2	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
15	C	o_3	$\mathbf{o_2}$	$\mathbf{o_2}$	o_2
8	D	o_4	o_4	o_1	o_4
7	E	o_3	o_3	o_1	$\mathbf{o_1}$

If the current winner is o, then agent i will switch its vote to some candidate o' provided

1. i prefers o' to o, and
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Size	Group	I	II	III	IV	V	VI	VII	$\cdots$
40	A	o_1	o_1	o_2	o_2	o_2	o_1	o_1	o_1
30	B	o_2	o_3	o_3	o_2	o_2	o_2	o_3	o_3
30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

Example III

$$P_A^* = (o_1, o_2, o_3)$$
$$P_B^* = (o_2, o_3, o_1)$$
$$P_C^* = (o_3, o_1, o_2)$$

Size	Group	I	II	III	IV	V	VI	VII	$\cdots$
40	A	o_1	o_1	o_2	o_2	o_2	o_1	o_1	o_1
30	B	o_2	o_3	o_3	o_2	o_2	o_2	o_3	o_3
30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

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30	B	o_2	o_3	o_3	o_2	o_2	o_2	o_3	o_3
30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

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Size	Group	I	II	III	IV	V	VI	VII	...
40	A	o_1	o_1	o_2	o_2	o_2	o_1	o_1	o_1
30	B	o_2	o_3	o_3	o_2	o_2	o_2	o_3	o_3
30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

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Size	Group	I	II	III	IV	V	VI	VII	...
40	A	o_1	o_1	o_2	o_2	o_2	o_1	o_1	o_1
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30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

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Size	Group	I	II	III	IV	V	VI	VII	$\cdots$
40	A	o_1	o_1	o_2	o_2	o_2	o_1	o_1	o_1
30	B	o_2	o_3	o_3	o_2	o_2	o_2	o_3	o_3
30	C	o_3	o_3	o_3	o_3	o_1	o_1	o_1	o_3

Summary, so far

Agents, knowing an aggregation function, will strategize if they know

a. enough about other agents' preferences and
b. that the output of the aggregation function of a changed preference will provide them with a more favorable result.

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Thank You!