Introduction to Formal Epistemology

Lecture 1

Eric Pacuit and Rohit Parikh

August 13, 2007

Lecture 1: Introduction, Motivation and Basic Models of Knowledge

Lecture 2: Knowledge in Groups and Group Knowledge

Lecture 3: Reasoning about Knowledge and ....

Lecture 4: Logical Omniscience and Other Problems

Lecture 5: Reasoning about Knowledge in the Context of Social Software

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Course Website:

staff.science.uva.nl/~epacuit/formep_esslli.html

Reading Material: The course reader and references therein.

- Introduction and Motivation

- Epistemic Logic

What do we want?

A simple mathematical model that *faithfully represents* (the agents information in) social interactive situations.

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J. Hintikka. *Knowledge and Belief*. 1962, recently republished.

See references in the notes!

Single-Agent Epistemic Logic: The Language

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  - "It is raining"
  - "The talk is at 2PM"
  - "The card on the table is a 7 of Hearts"

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- The usual definitions for $\rightarrow, \vee, \leftrightarrow$ apply
- Define $L\varphi$ as $\neg K \neg\varphi$

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$L\varphi$: "φ is an epistemic possibility"

$KL\varphi$: "Ann knows that she thinks φ is possible"

Single-Agent Epistemic Logic: Kripke Models

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- $V : \mathrm{At} \rightarrow \wp(W)$ is a valuation function assigning propositional variables to worlds

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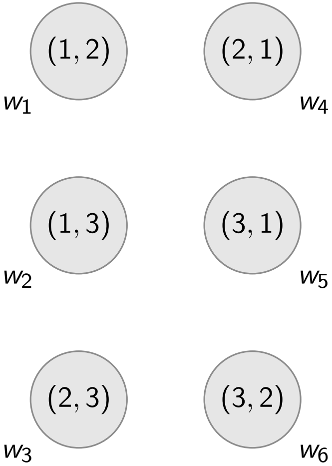

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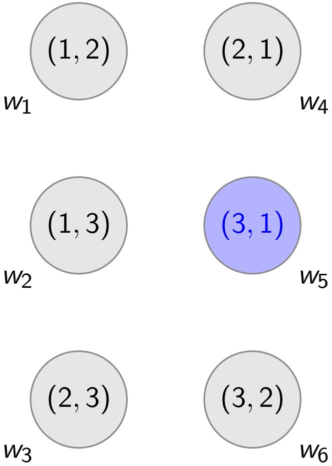

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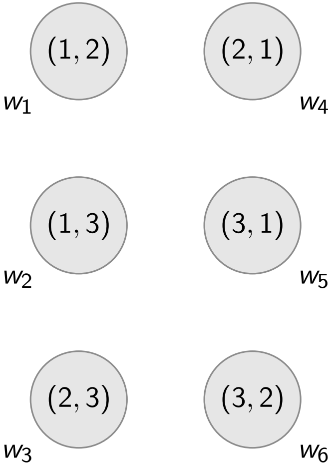

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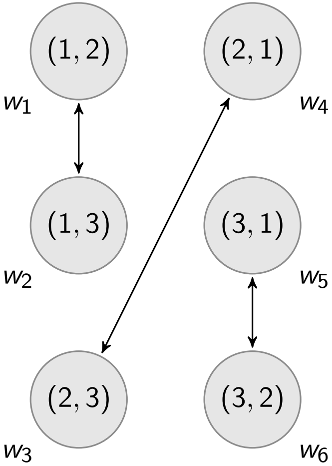

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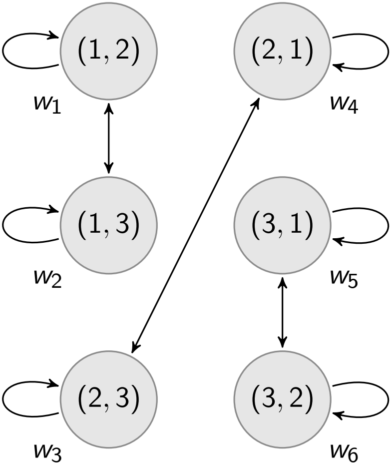

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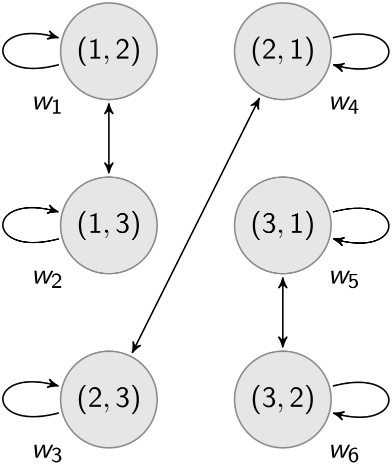
Suppose H_i is intended to mean "Ann has card i"

T_i is intended to mean "card i is on the table"

Eg., $V(H_1) = \{w_1, w_2\}$



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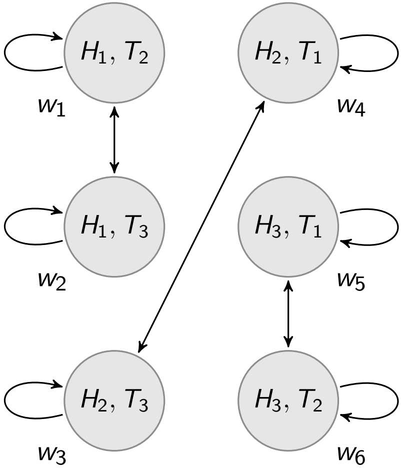

Single Agent Epistemic Logic: Truth in a Model

Given $\varphi \in \mathcal{L}$, a Kripke model $\mathcal{M} = \langle W, R, V \rangle$ and $w \in W$

$\mathcal{M}, w \models \varphi$ means "in $\mathcal{M}$, if the actual state is w, then φ is true"

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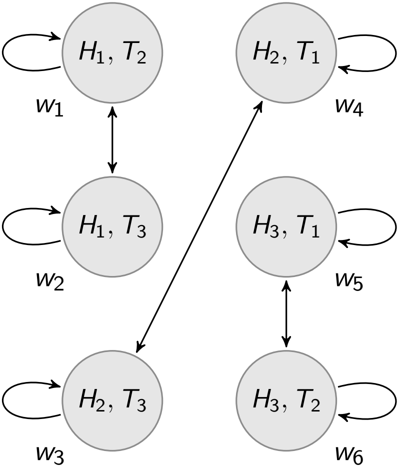

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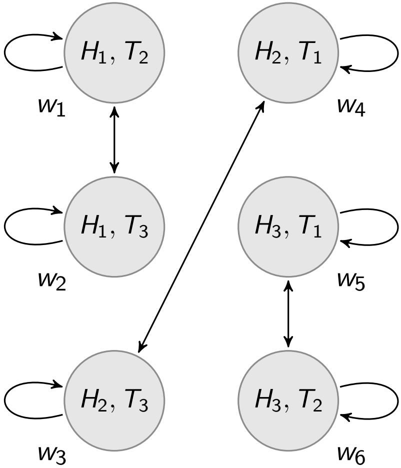

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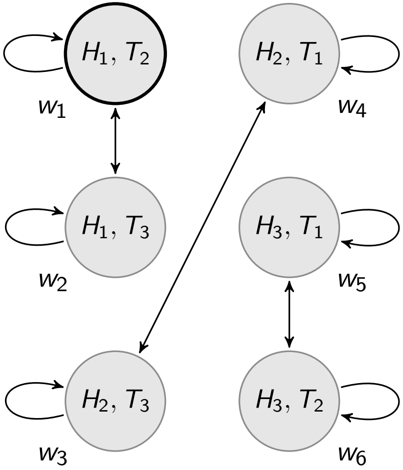

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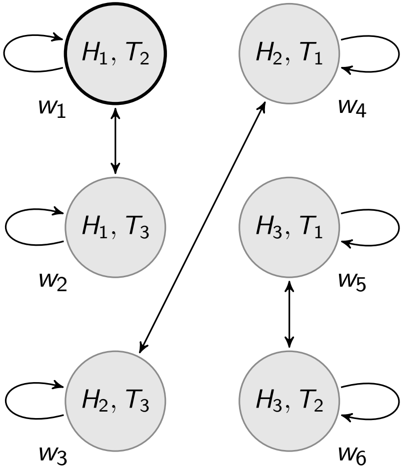

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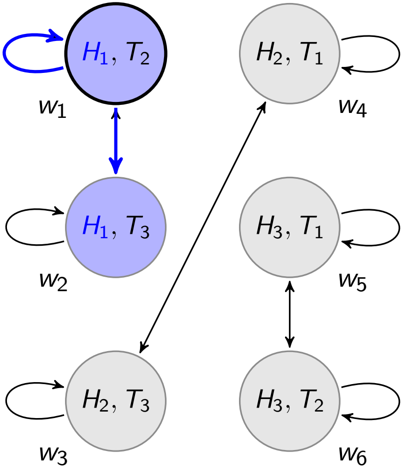

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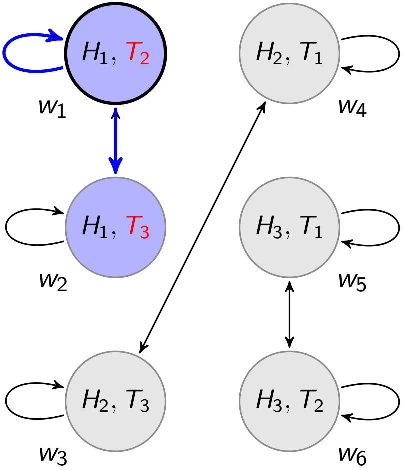

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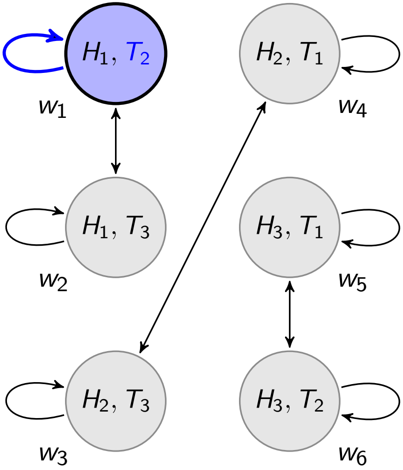
$\mathcal{M}, w_1 \models LT_2$



Some Questions

Should we make additional assumptions about R (i.e., reflexive, transitive, etc.)

What idealizations have we made?

Some Notation

A Kripke Frame is a tuple $\langle W, R\rangle$ where $R \subseteq W \times W$.

φ is valid in a Kripke model $\mathcal{M}$ if $\mathcal{M}, w \models \varphi$ for all states w (we write $\mathcal{M} \models \varphi$).

φ is valid on a Kripke frame $\mathcal{F}$ if $\mathcal{M} \models \varphi$ for all models $\mathcal{M}$ based on $\mathcal{F}$.

Logical Omniscience

Fact: φ is valid then $K\varphi$ is valid

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Fact: $K\varphi \wedge K\psi \rightarrow K(\varphi \wedge \psi)$ is valid on all Kripke frames

Logical Omniscience

Fact: If $\varphi \rightarrow \psi$ is valid then $K\varphi \rightarrow K\psi$ is valid

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Fact: $K(\varphi \rightarrow \psi) \rightarrow (K\varphi \rightarrow K\psi)$ is valid on all Kripke frames.

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Fact: $\varphi \leftrightarrow \psi$ is valid then $K\varphi \leftrightarrow K\psi$ is valid

Correspondence

Definition

A model formula φ corresponds to a property P (of a relation in a Kripke frame) provided

$$\mathcal{F} \models \varphi \text{ iff } \mathcal{F} \text{ has } P$$

Modal Formula	Corresponding Property
$K\varphi \rightarrow \varphi$	Reflexive
$K\varphi \rightarrow KK\varphi$	Transitive
$\neg K\varphi \rightarrow K\neg K\varphi$	Euclidean
$\varphi \rightarrow KL\varphi$	Symmetric
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Nec	$\dfrac{\varphi}{K\psi}$

Theorem

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- $\mathcal{M}, w \models K_i\varphi$ if for each $v \in W$, if wR_iv, then $\mathcal{M}, v \models \varphi$

Multi-agent Epistemic Logic

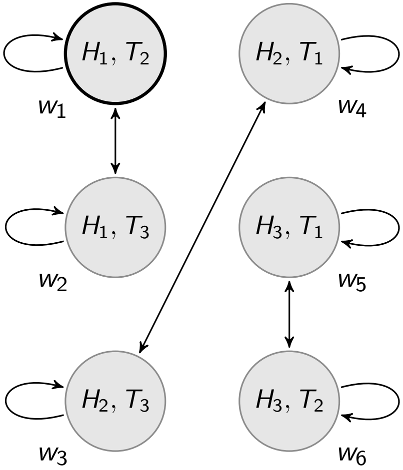
- $K_A K_B \varphi$: "Ann knows that Bob knows φ"
- $K_A(K_B \varphi \vee K_B \neg\varphi)$: "Ann knows that Bob knows whether φ
- $\neg K_B K_A K_B(\varphi)$: "Bob does not know that Ann knows that Bob knows that φ"

Example

Suppose there are three cards: 1, 2 and 3.

Ann is dealt one of the cards, one of the cards is placed face down on the table and the third card is put back in the deck.

Suppose that Ann receives card 1 and card 2 is on the table.



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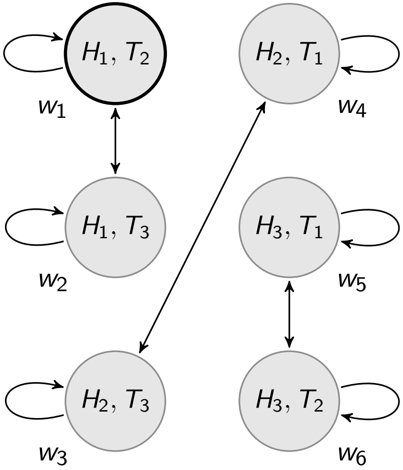

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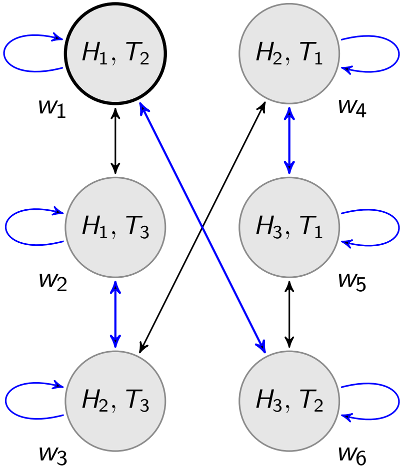

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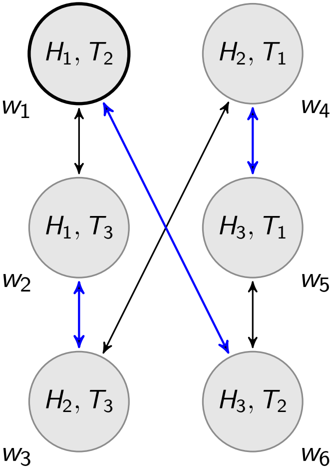

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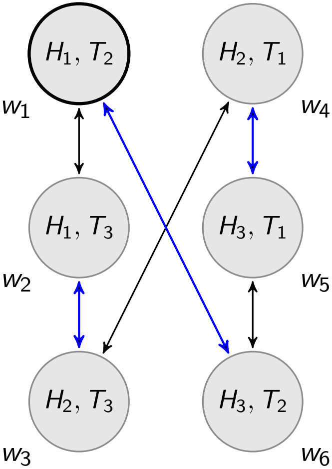
$\mathcal{M}, w \models K_B(K_A H_1 \vee K_A \neg H_1)$



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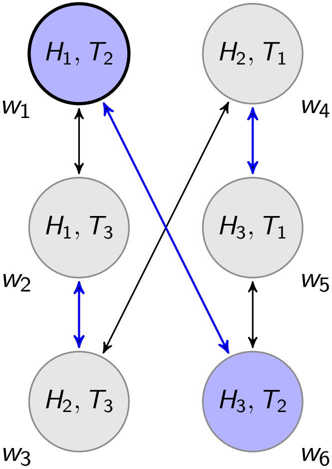

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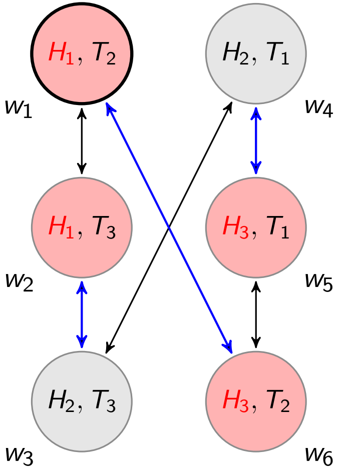

Example

Ann would like Bob to attend her talk; however, she only wants Bob to attend if he is interested in the subject of her talk, not because he is just being polite.

There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

Is this procedure correct?

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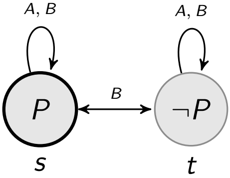
There is a very simple procedure to solve Ann's problem: *have a (trusted) friend tell Bob the time and subject of her talk.*

Is this procedure correct? Yes, if

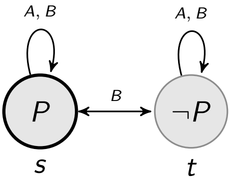
1. Ann knows about the talk.
2. Bob knows about the talk.
3. Ann knows that Bob knows about the talk.
4. Bob *does not* know that Ann knows that he knows about the talk.
5. *And nothing else.*

Example



P means "The talk is at 2PM".

Example



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$\mathcal{M}, s \models K_A P \wedge \neg K_B P$

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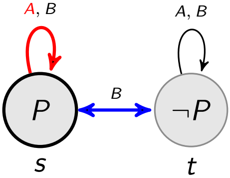

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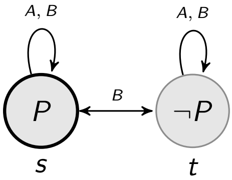

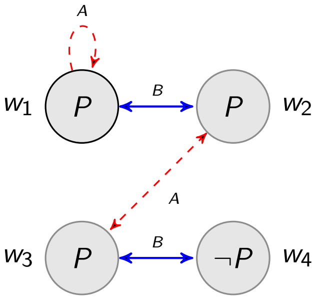

Thank you!